Prospect Theory and Trading Patterns

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Abstract

Reference dependence, loss aversion, and risk seeking for losses together comprise the preference-based component of prospect theory that sets its value function apart from the standard risk-aversion model. Using an elasticity analysis, we show that this distinctive preference component serves to underpin negative-feedback trading propensities, but cannot manifest itself in behavior directly or holistically at the individual-choice level. We then propose and demonstrate that the market interaction between prospect-theory investors and regular CRRA investors allows this preference component to dominate in equilibrium behavior and hence helps to reestablish the intuitive link between prospect-theory preferences and negative-feedback trading patterns. In the model, the interaction also reconciles the contrarian behavior of prospect-theory investors with asymmetric volatility and short-term return reversal. The results suggest that prospect-theory preferences can lead investors to behave endogenously as contrarian noise traders in the market interaction process.

Key words: Prospect theory; Negative-feedback trading; Price elasticity of demand; Contrarian behavior; The disposition effect; Noise trading

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1. Introduction

For a variety of reasons, prospect theory (Kahneman and Tversky, 1979; henceforth, PT) has become a major hypothesis for individual behavior in economic analysis. In particular, many scholars have referred to the S-shaped value function from PT as a leading preference-based explanation for negative-feedback trading patterns including short-term contrarian behavior (buying after prices decrease and selling after prices increase) and the disposition effect (individuals are more likely to sell nominal winners than losers),¹ both of which are supported by a substantial body of evidence (e.g., Barber and Odean, 2013). However, some recent theoretical studies (Barberis and Xiong, 2009; Hens and Vlcek, 2011) find counterintuitive results, indicating that the value function of PT as a whole does not necessarily lead to trading behavior that is consistent with the disposition effect in formal portfolio choice models. In this paper, we suggest a decomposition approach to the implications of PT’s value function and its components to understand how investors trade in response to price changes, and we further show that PT preference can yield negative-feedback trading patterns at a market-interaction level, despite failing to do so at the individual-choice level.

Our decomposition is motivated by the statement of Kahneman (2011, 288) that “prospect theory was accepted by many scholars ... because the concepts that it added to utility theory ... were worth the trouble; they yield new predictions that turned out to be true.” Specifically, although the value function favors risk aversion in the domain of gains, it deviates from the standard risk-aversion model (e.g., based on its gain function) in three distinct ways: (1) reference dependence (framing a decision problem around a reference point), (2) loss aversion (overweighting losses

¹As noted by Grinblatt and Keloharju (2000, 2001), the disposition effect can be easily interpreted as contrarian behavior with respect to price changes.
with respect to comparable gains), and (3) risk seeking for losses. For brevity, we refer to these combined characteristics as PT’s *loss aversion component*. However, few studies have examined this preference component in isolation from (conventional) risk aversion. This lack of research raises questions about whether the loss aversion component of PT has the capability of yielding negative-feedback trading propensities, and when it manifests itself in behavior. Moreover, trading actions are outcomes in exchange relationships. Such actions are not just autonomous at the individual level, but also a derivative of the interaction process between different market participants. Another question that has yet to be answered is whether market interaction helps to reestablish the link between PT and negative-feedback trading behavior.

We begin with the case of individual choice by conducting an elasticity analysis. We interpret the CRRA (Constant Relative Risk Aversion) utility function in the value function, i.e., its gain part, and the deviation of the value function from the benchmark CRRA function as the manifestations of PT’s risk and loss aversion components, respectively. The elasticity technique allows us to explicitly decompose the variation in a PT investor’s stock holdings (i.e., the investor’s trading behavior) due to price changes into the contributions from the two preference-based components. We find that the risk aversion component is generally characterized by a positive constant elasticity, while the loss aversion component is mainly characterized by state-dependent negative elasticities. The risk aversion component can therefore be interpreted as a source of the positive-feedback wealth (or portfolio-rebalancing) effect, and the loss aversion component can be considered a source of psychological motives in favor of negative-feedback trading.

We further show that the loss aversion component can become largely irrelevant in determining PT investor’s trading patterns under certain circumstances. Under the standard parametric specification of PT suggested by Tversky and Kahneman (1992), the value function is flat in the gain domain and hence the risk aversion component becomes close to risk neutral. The very low risk aversion then yields a prominent positive-feedback trading propensity in the portfolio-choice context with reasonable financial parameters. So, although the loss aversion component continues to
favor negative-feedback trading, the risk aversion component overcomes this effect and produces a
relation between PT preference and positive-feedback trading. This relation gradually breaks down
as the value function becomes more concave for gains. Accordingly, the partial reflection hypothe-
sis (i.e., utility is rather concave for gains, but mildly convex for losses; see, e.g., Loewenstein and
Prelec, 1992; Wakker et al., 2007) becomes a possible candidate for delivering negative-feedback
trading patterns at the level of individual choice, although still in a different way from that sug-
gested by the loss aversion component.

We then consider a market interaction case by extending the previous partial equilibrium anal-
ysis into the general equilibrium case in an economy consisting of two types of investors: PT and
regular CRRA investors. The equilibrium results reveal the dominance of the loss aversion com-
ponent in the form of preference heterogeneity. As a consequence, the negative-feedback trading
propensity of PT investors becomes reliably prominent, even when we use the parameter values ob-
tained by Tversky and Kahneman (1992). Our analysis also suggests that PT’s negative-feedback
trading actions are associated with price pressures that are consistent with the asymmetric volatility
effect (Black, 1976; Glosten et al., 1993; Avramov et al., 2006; Hibbert et al., 2008) and short-term
return reversals (Jegadeesh, 1990; Lehmann, 1990). The properties are consistent with the basic in-
tuition of the noise trading models (e.g., De Long et al., 1990; Campbell and Kyle, 1993; Campbell
et al., 1993) that noise traders’ demand shocks give rise to a source of volatility and price reversal
when demand curves are downward sloping. In this sense, because of the non-informational trad-
ing reasons introduced by the loss aversion component, PT investors can behave endogenously as
contrarian noise traders in the market interaction process.

With further analysis, we evaluate the validity and general applicability of our elasticity (sen-
sitivity) results. We test whether our basic conclusions are still valid when we measure the dispo-
sition effect with the method used by Odean (1998), as was done by Barberis and Xiong (2009).
In doing so, we complement the literature on the disposition effect by calibrating our PT portfo-
lio choice model with the seasonality of trading characteristics reported by Odean (1998). The
calibration yields results that are consistent with a partial reflection hypothesis.

This paper relates to the literature on PT in financial analysis. Of the notable works in this area, ours is most related to those of Barberis and Xiong (2009); Berkelaar et al. (2004); Berkelaar and Kouwenberg (2009). From a technical point of view, this study can be considered an extension of the portfolio choice and asset pricing models in Berkelaar et al. (2004); Berkelaar and Kouwenberg (2009) to a stock-holding elasticity (or sensitivity) analysis. The individual-choice case in the current work can be thought of in part as a continuous-time analogy of Barberis and Xiong (2009). We use a continuous-time market setting and elasticity concepts to yield a rigorous understanding of how different preference-based components of PT give rise to different trading propensities. This allows us to shed more light on both the power and limits of PT in the analysis of trading behavior. Moreover, to our knowledge, no study formally investigates the possibility that market interaction helps to reestablish the link between PT and negative-feedback trading behavior, or the possibility that PT contributes to reconciling contrarian behavior with noise trading. In addition, the return reversal prediction in our market-interaction case is contrary to the argument of Grinblatt and Han (2005) that by generating trading actions consistent with the disposition effect, PT preferences lead to momentum in stock returns. We thus have a different model and different results from theirs in terms of the nature of contrarian behavior or the implications of PT preferences for price and trade dynamics.

The remainder of this paper is organized as follows. In Section 2, we build stock demand (holding) as a function of stock price based on a continuous-time portfolio model, and analyze its price elasticities to examine how PT predicts trading behavior at the individual-choice level. In Section 3, we extend the previous partial-equilibrium setting to a general-equilibrium setting, and investigate how the market interaction between CRRA and PT investors determines investor and price behavior. In Section 4, we verify that the obtained results can also be extended very readily to the case of the disposition effect defined by the empirical methodology of Odean (1998), and offer an empirical calibration based on data from the literature. Section 5 concludes the paper.
2. Elasticity decomposition, trading patterns, and PT

2.1. Basic setup

This section uses a partial equilibrium complete market framework to analyze trading pattern issues. The framework can be considered the standard continuous-time analogy used by Barberis and Xiong (2009). There are two assets. The first asset is a riskless bond with a constant interest rate, $r_f$, while the second asset is a risky stock whose price $S(t)$ satisfies

$$\frac{dS(t)}{S(t)} = \mu dt + \sigma dB(t), \quad S(0) = S_0,$$

where both parameters $\mu$ and $\sigma > 0$ are constants and $B(t)$ is a standard Brownian motion. As suggested by Barberis and Xiong (2009), this one risky asset assumption is reasonable in a multi-stock setting when the investor is prone to “mental accounting,” or when the initial wealth is interpreted as the maximum amount the investor is willing to lose in any one stock.

In this setting, we can obtain the following unique state price density process:

$$d\zeta(t) = -\zeta(t) \left( r_f dt + \theta dB(t) \right), \quad \zeta(0) := \zeta_0,$$

where $\theta = \sigma^{-1}(\mu - r_f)$ denotes the market price of risk (or the Sharpe ratio).

Let $W(t)$ be the wealth of the investor at time $t$ and $H(t)$ be the number of stock shares held by the investor. The corresponding wealth process is then governed by

$$dW(t) = (W(t) - H(t)S(t))r_f dt + H(t)dS(t),$$

with $W(0) = W_0 > 0$ being the initial wealth of the investor.

Tversky and Kahneman (1992) formulate the PT’s value function as follows:

$$v(x) = \begin{cases} 
-\lambda(-x)^{\alpha_1} & \text{for } x < 0, \\
 x^{\alpha_2} & \text{for } x \geq 0,
\end{cases}$$

where $x$ represents the gain or loss relative to the status quo, $\lambda > 1$ is the coefficient of loss aversion, and both $\alpha_1$ and $\alpha_2 \in (0, 1)$ are the curvature parameters of utility. Using the experimental data,
Tversky and Kahneman (1992) estimate $\alpha_1 = \alpha_2 = 0.88$ and $\lambda = 2.25$. Following the conventions in the literature (e.g., Benartzi and Thaler, 1995; Berkelaar et al., 2004; Barberis and Xiong, 2009), we omit the probability weighting function of PT and use the following value function to describe PT preferences:

$$U(W) = \begin{cases} 
-\lambda(W - \bar{W})^{\alpha_1} & \text{for } W < \bar{W}, \\
(W - \bar{W})^{\alpha_2} & \text{for } W \geq \bar{W}, 
\end{cases}$$

(5)

where $\bar{W}$ is the (exogenous) reference wealth level.

The PT investor’s optimal stock demand is then given by the solution to the following optimization problem:

$$\max_{H(t)} \mathbb{E}[U(W(T))], \quad \text{s.t. } W(t) \geq 0, \ \forall \ t \in [0, T], \quad \text{(PT)}$$

where $T$ denotes the time horizon.

Note that this formulation nests the CRRA case. For example, when $\bar{W} = 0$, the loss aversion component never takes effect, and Problem (PT) collapses into a standard risk-aversion model with CRRA, i.e.,

$$\max_{H(t)} \mathbb{E}[W(T)^{\alpha_2}], \quad \text{s.t. } W(t) \geq 0, \ \forall \ t \in [0, T]. \quad \text{(RA)}$$

As stated in the Introduction, Problem (RA) thus serves naturally as a benchmark of Problem (PT) for evaluating PT’s decision-making process. In the following analysis, the parameter $\alpha_2$ in Problem (RA) is kept the same as in Problem (PT).

Following the definition of point elasticity in microeconomics, we formulate the price elasticity of stock demand as

$$\varepsilon(t) = \frac{\partial H(t)}{\partial S(t)} \frac{S(t)}{H(t)} = \frac{\partial \ln H(t)}{\partial \ln S(t)}. \quad \text{(6)}$$

The idea that $\varepsilon(t)$ quantifies the demand response rate with respect to a price change can be further explained using a form of stochastic differential equation. Using the Itô formula, we have

$$d \ln H(t) = \frac{\partial \ln H(t)}{\partial \ln S(t)} d \ln S(t) + \left[ \frac{\partial \ln H(t)}{\partial t} + \frac{1}{2} \frac{\partial^2 \ln H(t)}{\partial (\ln S(t))^2} \sigma^2 \right] dt. \quad \text{(6.1)}$$

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It is clear that $\varepsilon(t)$ represents the sensitivity of $d \ln H(t)$ to $d \ln S(t)$, hence the sensitivity of $dH(t)/H(t)$ to $dS(t)/S(t)$; therefore, it assesses the effect of a differential change in the stock price on the stock demand $H(t)$.

Elasticity analysis is an intuitively appealing way to identify patterns in trading behavior. A negative-feedback (positive-feedback) trading activity can be characterized by a negative (positive) price elasticity. Accordingly, the prominence of the negative-feedback trading tendency can be measured by the (unconditional) probability of the negative-elasticity case. Thus, if negative feedback occurs more frequently compared with positive feedback, i.e.,

$$\Pr(\varepsilon(t) < 0) > 0.5,$$

we can conclude that the investor is most inclined toward a negative-feedback trading propensity and hence tends to be a contrarian investor at time $t$. Because price changes directly determine an investor’s gains or losses from stock purchases, the negative-feedback condition also speaks directly to the disposition effect.\(^2\) Likewise, if $\Pr(\varepsilon(t) < 0) < 0.5$, we would anticipate that the investor is more likely to be subject to positive-feedback trading propensities, and hence tends to be a momentum investor at time $t$.

We denote the time-$t$ stock demand (i.e., share holding) of Problem (PT) with $H^{PT}(t)$, and that of Problem (RA) with $H^{RA}(t)$. The gap between $H^{PT}(t)$ and $H^{RA}(t)$ is then determined by the deviation of PT’s value function from the benchmark CRRA utility function, which we refer to as the loss aversion component of PT. Accordingly, we adopt the ratio

$$\delta(t) = \frac{H^{PT}(t)}{H^{RA}(t)}$$

(7)

to quantify the extent to which the loss aversion component contributes to the stock demand of a

\(^2\)One may argue that this condition takes the most recent price as the reference price and hence cannot explain other aspects of the disposition effect discussed in the literature. Section 4 offers more details by basing the reference point on the average purchase price.
PT investor. Based on these definitions, we can obtain

\[ \varepsilon_{PT}(t) = \varepsilon^{RA}(t) + \varepsilon^{LA}(t), \]  

(8)

where

\[ \varepsilon_{PT}(t) = \frac{\partial \ln H_{PT}(t)}{\partial \ln S(t)}, \quad \varepsilon^{RA}(t) = \frac{\partial \ln H^{RA}(t)}{\partial \ln S(t)}, \quad \varepsilon^{LA}(t) = \frac{\partial \ln \delta(t)}{\partial \ln S(t)}. \]

Eq. (8) states that the price elasticity of \( H_{PT}(t) \) can be separated into two components. The first term, \( \varepsilon^{RA}(t) \) (the price elasticity of \( H^{RA}(t) \)), represents the price effect derived from the risk aversion component; and the other term, \( \varepsilon^{LA}(t) \) (the price elasticity of \( \delta(t) \)), represents the price effect derived from the loss aversion component.

2.2. Elasticity analysis

The following proposition establishes that the stock demand of the benchmark CRRA investor has a constant elasticity with respect to the stock price.

**Proposition 1.** For Problem (RA), we have the following conclusions:

(i) The investor’s time-\( t \) stock demand is

\[ H^{RA}(t) = \frac{\theta}{(1 - \alpha_2)\sigma} D_h(t, y^{RA}) S(t)^{\frac{\theta}{1 - \alpha_2} - 1} > 0, \]

where

\[ D_h(t, x) = \left( \frac{\alpha_2}{x} \right)^{1/(1 - \alpha_2)} e^{\Gamma(t)} \exp \left\{ -\frac{1}{1 - \alpha_2} \left( \frac{\theta - \sigma^2}{2} - r_f - \theta \right) t \right\}, \]

\[ y^{RA} = \alpha_2 \left( \frac{\epsilon^{(0)}(t)}{W_0} \right)^{1 - \alpha_2}, \quad \Gamma(t) = \frac{\alpha_2}{1 - \alpha_2} \left( r_f + \frac{\theta}{2} \right) (T - t) + \frac{1}{2} \left( \frac{\alpha_2}{1 - \alpha_2} \right)^2 \theta^2 (T - t). \]

(ii) The price elasticity of the investor’s stock demand is

\[ \varepsilon^{RA}(t) = \varepsilon^{LA} = \frac{\theta}{(1 - \alpha_2)\sigma} - 1. \]  

(9)

**Proof:** See the Appendix.
Eq. (9) shows that the price elasticity is independent of the stock prices, and that its magnitude is positively related to \( \theta \) (or equity premium \( \mu - r_f \)) and \( \alpha_2 \). Recall the classic works of Samuelson (1969) and Merton (1969), who derive the following optimal constant proportion portfolio strategy for CRRA investors:

\[
\pi(t) = \pi^* = \frac{\theta}{(1 - \alpha_2)\sigma}.
\]

Thus \( \varepsilon^{RA} \) directly reflects trading propensities that are required to maintain constant portfolio weights, i.e., the so-called “portfolio-rebalancing” effect. Note that \( \varepsilon^{RA} \) can be either negative or positive, depending on whether \( \pi^* < 1 \) or \( \pi^* > 1 \), i.e., whether \( \alpha_2 < 1 - \theta/\sigma \) or \( \alpha_2 > 1 - \theta/\sigma \).

Odean (1998) offers a hypothesis that the portfolio rebalancing predicted by CRRA preferences could produce the disposition effect, which corresponds to the case \( \alpha_2 < 1 - \theta/\sigma \) (i.e., the CRRA investor should be risk averse enough).

However, because the standard specification of PT requests \( \alpha_2 \in (0, 1) \), a high equity premium implies that the case \( \alpha_2 < 1 - \theta/\sigma \) occurs here infrequently, if not seldom. Indeed, the high equity premium is a robust empirical phenomenon known as the so-called “equity premium puzzle” (Mehra and Prescott, 1985). In particular, Barberis and Xiong (2009) argue that low equity premium cases have little intrinsic theoretical or empirical interest because without a large enough equity premium, PT investors would not buy any stock at the initial time. Considering these facts, we draw the following conclusion.

**Remark 1.** \( \varepsilon^{RA} \) is typically a positive constant with reasonable financial parameters.

For example, when \( \sigma = 0.194, \theta = 0.303, \) and \( \alpha_2 = 0.88, \) we have

\[
\varepsilon^{RA} \approx 12. \tag{10}
\]

This elasticity value characterizes a prominent positive-feedback trading tendency.

The following proposition characterizes the optimal solution of Problem (PT).

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Proposition 2. (i) The optimal terminal wealth of a PT investor is

\[ W^{PT}(T) = \begin{cases} 
W + \left(\frac{\gamma^{PT}(T)}{\alpha_2}\right)^{1/(\alpha_2-1)} & \text{if } \zeta(T) < \bar{\zeta}, \\
0, & \text{if } \zeta(T) \geq \bar{\zeta},
\end{cases} \]

where \( \bar{\zeta} \) is the solution of the equation

\[ f(x) = \frac{1 - \alpha_2}{\alpha_2} \left( \frac{1}{y^{PT}x} \right)^{\alpha_2/(1-\alpha_2)} (\alpha_2^{1/(1-\alpha_2)} - \frac{\bar{W}y^{PT}x + \lambda\bar{W}^{\alpha_1}}{\alpha_2^{1/(1-\alpha_2)}} - \frac{Wy^{PT}x}{\alpha_2} - \frac{\lambda W^{\alpha_1}}{\alpha_2}) = 0 \] (12)

and the multiplier \( y^{PT} \) satisfies \( E[\zeta(T)W^{PT}(T)] = \zeta_0 W_0 \).

(ii) For this terminal wealth profile, an increase of the parameter \( \lambda \) or the initial wealth \( W_0 \) leads to an increase in the breakpoint \( \bar{\zeta} \).

Proof: The proof can be found in Berkelaar et al. (2004).

Proposition 3 presents the explicit expressions of \( H^{PT}(t), \delta(t), \) and \( \varepsilon^{LA}(t) \) as functions of \( S(t) \).

Proposition 3. (i) The PT investor’s stock demand is

\[ H^{PT}(t) = H_1^{PT}(t) + H_2^{PT}(t) + H_3^{PT}(t) > 0, \]

where

\[ H_1^{PT}(t) = \frac{\phi(q_1(\bar{S}, t))}{\sigma \sqrt{T - t}} W e^{-r_f(T-t)S(t)^{-1}} > 0, \]
\[ H_2^{PT}(t) = \frac{\eta \Phi(q_2(\bar{S}, t))}{(1 - \alpha_2)\sigma} D_h(t, y^{PT}S(t) e^{-\alpha_2/2})^{-1} > 0, \]
\[ H_3^{PT}(t) = \frac{\phi(q_3(\bar{S}, t))}{\sigma \sqrt{T - t}} D_h(t, y^{PT}S(t) e^{-\alpha_2/2})^{-1} > 0, \]

\( \bar{S} \) is the stock price corresponding to \( \bar{\zeta} \) (see Proposition 2), \( \bar{\zeta} \) the function \( D_h(t, x) \) can be found in Proposition 1, and

\[ q_1(x, t) = \frac{(r_f - \theta^2/2)(T - t)}{\theta \sqrt{T - t}} + \frac{\ln S(t) - \ln x}{\sigma \sqrt{T - t}}, \quad q_2(x, t) = q_1(x, t) + \frac{\theta \sqrt{T - t}}{1 - \alpha_2}. \]

\[ \footnote{The point \( \bar{S} \) can be calculated from Eq. (A.4) in the Appendix.} \]
(ii) The ratio of $H_{PT}(t)$ to $H_{RA}(t)$ is

$$
\delta(t) = \delta_1(t) + \delta_2(t) + \delta_3(t) > 0,
$$

where

$$
\delta_1(t) = \frac{H_{PT}(t)}{H_{RA}(t)} = \frac{1 - \alpha_2}{\theta \sqrt{T - t}} \phi(q_1(S, t)) \frac{\bar{W} e^{-r_f(T-t)} D_{\theta}(t, y_{RA}) S(t)^{-\frac{\theta}{(T-t)^{\theta}}}}{> 0},
$$

$$
\delta_2(t) = \frac{H^2_{PT}(t)}{H_{RA}(t)} = \Phi(q_2(S, t)) \left(\frac{y_{RA}}{\theta^{\theta}}\right)^{\frac{1}{\theta}} > 0,
$$

$$
\delta_3(t) = \frac{H^3_{PT}(t)}{H_{RA}(t)} = \frac{1 - \alpha_2}{\theta \sqrt{T - t}} \phi(q_2(S, t)) \left(\frac{y_{RA}}{\theta^{\theta}}\right)^{\frac{1}{\theta}} > 0.
$$

(iii) The price elasticity of $\delta(t)$ is

$$
\varepsilon^{LA}(t) = - (\ln S(t) - \ln S_{\delta}(t)) \frac{\delta_1(t) + \delta_3(t)}{\delta_1(t) - \delta_3(t)} \sigma^2(T - t), \tag{13}
$$

where

$$
S_{\delta}(t) = S \exp \left\{ - \left[ \frac{r_f \sigma^2}{\theta} + (\mu - r_f) \left( \frac{1}{1 - \alpha_2} \frac{\delta_1(t)}{\delta_1(t) + \delta_3(t)} - \frac{1}{2} \right) \right] (T - t) \right\}.
$$

(iv) The critical point $\bar{S} = S_{\delta}(T)$ decreases, and hence the likelihood of the region $[S(T) > \bar{S}_{\delta}(T)]$ increases, as $\lambda$ increases, $W_0$ increases, or $\bar{W}$ decreases.

**Proof:** See the Appendix.

Eq. (13) shows that the sign of the price elasticity $\varepsilon^{LA}(t)$ is determined by the level of the stock price. In the region above the critical stock price, i.e.,

$$
S(t) > \bar{S}_{\delta}(t), \tag{14}
$$

$\varepsilon^{LA}(t)$ is negative, suggesting that the loss aversion component yields negative-feedback trading propensities. In this case, the loss aversion component gives rise to a perceptual and psychophysical need for trading against recent price changes.

Otherwise, when $S(t) < \bar{S}_{\delta}(t)$, a sign reversal occurs and $\varepsilon^{LA}(t)$ becomes positive. Eq. (11) shows that the terminal outcomes of the portfolio investment are assessed as pure gains in all good
states with $\zeta(T) < \bar{\zeta}$ (or $S(T) > \bar{S}$), and as pure losses in all bad states with $\zeta(T) > \bar{\zeta}$ (or $S(T) < \bar{S}$). Intuitively, in the region $[S(t) < \bar{S}_{\delta}(t)]$, it is very likely that the investor will end up in the pure-loss region $[S(T) < \bar{S}_{\delta}(T) = \bar{S}]$. That said, the risky stock is not anticipated to offer a chance to break even in this region. This makes the stock appear less attractive to the stockholder after its price declines further. So the loss aversion component induces positive-feedback trading propensities if the stock price falls below the critical value. As such, the likelihood of negative elasticities is primarily determined by the perceived opportunity to make money, i.e., $\Pr(S(T) > \bar{S})$. By this logic, we make the following remark.

**Remark 2.** The loss aversion component yields negative elasticities when the stock price is greater than the critical value $\bar{S}_{\delta}$, and positive ones when it is lower. Consistent with item (iv) in Proposition 3, the likelihood of negative elasticities, $\Pr(\varepsilon^{LA}(t) < 0)$, tends to increase as $\lambda$ increases or $\bar{W}$ decreases.

To offer a vivid illustration of these results, Figure 1 plots $\varepsilon^{LA}(t)$ as a function of $S(t)$. The small circles in the figure mark the positions of $\bar{S}_{\delta}$. The figure shows that the negative-elasticity case generally dominates; hence, the condition $\Pr(\varepsilon^{LA}(t) < 0) > 0.5$ can be easily satisfied. The case $\lambda = 2.25$ in Figure 1a depicts the parametric specification of the value function suggested by Tversky and Kahneman (1992). In this case, we can obtain $\bar{S}_{\delta}(t) = 0.65$, and $\Pr(\varepsilon^{LA}(t) < 0) = \Pr(S(t) > 0.65) \approx 1$,\(^4\) implying an extremely prominent negative-feedback trading propensity. Intuitively, when the equity premium (or the market price of risk) is rather high, the optimal strategy features a high probability of making money and accordingly the loss aversion component is primarily characterized by negative elasticities.

Figure 1 also illustrates the typical shape of $\varepsilon^{LA}(t)$ and shows that $\varepsilon^{LA}(t)$ is a non-monotonic function of the stock price. In the figure, $\varepsilon^{LA}(t)$ first declines with $S(t)$, then increases after reaching

\(^4\)Because $\mu$ and $\sigma$ are constants and the stock price follows a geometric Brownian motion, $\ln S(t)$ is normally distributed with mean $\ln S_0 + (\mu - \frac{1}{2} \sigma^2) t$ and variance $\sigma^2 t$.\)
its bottom, and finally converges to zero when \( S(t) \) is extremely high. The non-monotonic pattern is consistent with the intuition that because loss aversion (captured by \( \lambda \)) introduces a kink in the value function at the status-quo type reference point, substantial aversion to risk is concentrated in the intermediate region (where the stock price change is small). However, in the extremely good states where the stock price has already risen significantly, the value function becomes quite similar to its risk aversion component and hence the effect of the loss aversion component \( \epsilon^{LA}(t) \) is fairly close to zero.

As suggested in Remark 2, the curves in Figure 1a further show that the region \( [\epsilon^{LA}(t) < 0] \) grows at the expense of the region \( [\epsilon^{LA}(t) > 0] \) as \( \lambda \) increases, and those in Figure 1b show that the region \( [\epsilon^{LA}(t) < 0] \) grows as \( \overline{W} \) decreases. Because our primary concern is with the standard implementation of PT, we assume \( \overline{W} = W_0 e^{rT} \) in the following analysis, an assumption made by most of the literature. We also fix \( \alpha_1 = 0.88 \) in the following analysis because the evidence is rather mixed on the risk-seeking feature in the domain of losses (e.g., Kaustia, 2010).

Figure 2 further displays a sensitivity analysis for the probability that \( [\epsilon^{LA}(t) < 0] \) with respect to \( \lambda \) and \( \alpha_2 \). Again, Figure 2a shows that increasing \( \lambda \) has a positive causal effect on the negative-elasticity probability. In contrast, Figure 2b indicates that the effect of \( \alpha_2 \) on the negative probability of \( \epsilon^{LA}(t) \) is quite weak. The results confirm the salience of our treatments, which assume that the risk and loss aversion components concern different preference features. The curve wedges further illustrate that the negative-elasticity probability decreases with \( t \). This pattern is consistent with the seasonality observed by Odean (1998) where the disposition effect tends to decrease from January to December.

The figure also reveals that the negative-elasticity probability is generally higher than 0.5. In general terms, we thus make the following remark.

**Remark 3.** Under reasonable parameterizations of the value function, the loss aversion component is primarily characterized by negative elasticities.
2.3. What specification of PT’s value function favors the condition \( \varepsilon_{PT}(t) < 0 \)?

Up to this point, we have considered \( \varepsilon^{LA}(t) \) and \( \varepsilon^{RA} \) separately, and found them to be so diametrically opposed to each other when generating trading propensities. Now we confine our attention to the price elasticity of \( H^{PT}(t) \): \( \varepsilon^{PT}(t) = \varepsilon^{RA} + \varepsilon^{LA}(t) \). Clearly, the condition \( \varepsilon^{PT}(t) < 0 \) applies only to the states \( \varepsilon^{LA}(t) < -\varepsilon^{RA} \) where trading propensities are necessarily dominated by the loss aversion component.

In an extreme case, when \( \alpha_2 \) is close to one (and hence the risk aversion component is close to risk neutral), \( \varepsilon^{RA} \) has an extremely high value, as suggested by Eq. (9). That said, the sign of \( \varepsilon^{PT}(t) \) would be little influenced by the loss aversion component, although which has the capability of yielding negative-feedback trading propensities at an appropriate level. In this sense, the lack of curvature in the PT’s utility function for gains suggests that investors are more likely to exhibit positive-feedback trading behavior. This concern helps to further illustrate why the standard parametric specification offered by Tversky and Kahneman (1992), where \( \alpha_2 = 0.88 \), is not an appropriate explanation for the disposition effect in portfolio choice models, as demonstrated in Barberis and Xiong (2009). This counterintuitive result can then be interpreted as saying that it is not necessarily for PT investor’s trading behavior to be largely determined by the loss aversion component.

Figure 3 presents Iso negative-elasticity-probability lines, each representing a particular value of \( \Pr(\varepsilon^{PT}(t) < 0) \), in the \((\alpha_2, \lambda)\) coordinate system. The figure illustrates that \( \Pr(\varepsilon^{PT}(t) < 0) \) tends to increase as \( \alpha_2 \) decreases. The reason for this relationship is straightforward. Eq. (9) suggests that a decrease in \( \alpha_2 \) decreases \( \varepsilon^{RA} \) quickly. In contrast, Figure 2b shows that the effect of \( \alpha_2 \) on \( \varepsilon^{LA} \) is rather weak. In this manner, a lower \( \alpha_2 \) tends to make more “room” for the loss aversion component. We thus make the following remark.

**Remark 4.** As \( \alpha_2 \) decreases, the PT investor becomes more prone to the loss aversion component and accordingly is more likely to exhibit negative-feedback trading propensities.

In this sense, the partial reflection hypothesis (i.e., the value function is sufficiently concave
for gains, but mildly convex for losses) becomes a reasonable candidate for modeling negative-feedback trading patterns.\footnote{The mounting psychological evidence that confirms the partial reflection hypothesis can be found in Footnote 1 in Wakker et al. (2007).} Figure 3 confirms that the negative-feedback trading pattern of PT investors becomes fairly prominent when $\alpha_2$ is low enough.

Another intriguing result in Figure 3 is that the effect of $\lambda$ on the negative-elasticity probability becomes less clear as $\alpha_2$ decreases. In particular, in the top-left sections of the figure, we can observe that the negative-elasticity probability tends to decrease with $\lambda$, which is the opposite direction from that displayed in Figure 2a. Again, this difference between $\varepsilon^P(t)$ and $\varepsilon^{\lambda}(t)$ reminds us that PT’s value function and loss aversion component can be distinct in nature when applied to individual choices.

Although we focus on the standard implementation of PT based on a power utility function, it is possible that other basic utility assumptions may be used to make PT more readily predict a negative-feedback trading tendency. For example, Kyle et al. (2006) find that PT preferences predict trading actions that are consistent with a disposition effect by replacing the power function with the negative exponential function in the value function. From the perspective of this paper, the reason for this step is straightforward. As is well known, the negative exponential function, as a CARA (constant absolute risk aversion) utility model, lacks the wealth effect, which is closest in spirit to the portfolio-rebalancing effect contributed by the risk aversion component. Accordingly, such a CARA implementation of PT would lead more readily to a dominance of the loss aversion component.

3. Market interaction and equilibrium trading patterns

In this section, we invoke a market interaction view by integrating the portfolio choice problems from the previous section into an exchange economy. As is common in the equilibrium when investors have heterogeneous risk preferences, non-informational heterogeneity leads to heteroge-
neous trading behavior across investors. In this manner, the market interaction between PT and CRRA investors should lead to a dominance of the loss aversion component, as the basic source of their preference variation, in equilibrium behavior. A natural hypothesis thus is that this market interaction can reestablish the link between PT preferences and negative-feedback trading behavior.

We formally justify this thinking based on the pure-exchange general equilibrium model of Lucas (1978), adapted to a continuous-time economy by Basak (1995), and applied to analyze PT investors by Berkelaar and Kouwenberg (2009). We assume that the risky stock is in a fixed supply of 1, and the proportions of CRRA and PT investors are $\psi$ and $1 - \psi$ (both positive), respectively. By the market clearing mechanism, we have

$$\psi H_{e}^{eA}(t) + (1 - \psi) H_{e}^{ePT}(t) = 1, \quad (15)$$

where $H_{e}^{eA}(t)$ and $H_{e}^{ePT}(t)$ are the equilibrium stock holdings of the CRRA and PT investors, respectively. Suppose that the riskless bond is in zero net supply, and the risky stock pays dividends at a rate of $D(t)$ at time $t$, while $D(t)$ satisfies

$$dD(t) = \mu_D D(t) dt + \sigma_D D(t) dB(t), \quad (16)$$

where $\mu_D > 0$ and $\sigma_D > 0$ are constants. Further, assume that the investors’ utility of consumption $c(t)$ is modeled by a power utility function:

$$U_c(x) = \frac{1}{\gamma} x^{\gamma}, \quad \text{for } \gamma < 1. \quad (17)$$

The investor’s wealth process can then be formulated as

$$dW(t) = (W(t) - H(t)S(t))r_f(t)dt - c(t)dt + H(t) dS(t), \quad (18)$$

where each investor is endowed with initial wealth $W_0$.

In this setting, it is known from the studies of Basak (1995) and Berkelaar and Kouwenberg (2009) that an equilibrium exists with the equilibrium state price density

$$d\zeta(t) = -\zeta(t) \left( r_f dt + \theta dB(t) \right), \quad \zeta(0) = \zeta_0,$$
and

\[ r_f = (1 - \gamma)(\mu_D + 1/2(\gamma - 2)\sigma_D^2), \quad \theta = (1 - \gamma)\sigma_D. \]  \hfill (19)

Based on Problems (RA) and (PT), the individual optimization problem for each CRRA investor can then be formulated as

\[
\max_{c,H} \mathbb{E}\left[ \int_0^T U_c(c(s)) \, ds + \rho_1 W(T)^{\alpha_2} \right],
\]

\[
\text{s.t. } \mathbb{E}\left[ \int_0^T \zeta(s)c(s) \, ds + \zeta(T)W(T) \right] = \zeta(0)W_0,
\]

and the optimization problem for each PT investor can be formulated as

\[
\max_{c,H} \mathbb{E}\left[ \int_0^T U_c(c(s)) \, ds + \rho_2 U(W(T)) \right],
\]

\[
\text{s.t. } \mathbb{E}\left[ \int_0^T \zeta(s)c(s) \, ds + \zeta(T)W(T) \right] = \zeta(0)W_0,
\]

where \(\rho_1\) and \(\rho_2\) are the scaling terms to ensure that the wealth term dominates in the utility function. In comparison with the conventional investment problem (RA'), a unique characteristic of Problem (PT') is that its optimal time-\(T\) wealth may be zero (see Proposition 2). This characteristic would not lead to a market collapse because the existence of CRRA investors can guarantee that the stock would not have a zero value at time \(T\).\(^6\) Again, we can understand the initial wealth \(W_0\) of PT investors as the maximum amount they are willing to lose during their evaluation period.

From the solutions obtained by Berkelaar and Kouwenberg (2009), we can derive the equilibrium stock price in this paper as follows:

\[
S(t) = a(t)D(t) + \psi \nu_{y}^{(1-\gamma)/(1-\alpha_2)} \left( \frac{\nu_{y}^{\text{RA}}}{\rho_1} \right)^{1/(\alpha_2-1)} b(t)D(t)^{(1-\gamma)/(1-\alpha_2)} +
\]

\[
(1 - \psi) \left[ W e^{-r_f(T-t)} \Phi(d_1(D(t))) + \left( \frac{\rho_2 \alpha_2}{\gamma_{PT}} \right)^{(1/(1-\alpha_2))} e^{\Gamma(t)(\nu_{y}D(t))^{(1-\gamma)/(1-\alpha_2)}} \Phi(d_2(D(t))) \right],
\]

\(^6\)However, this characteristic means that extremely serious losses may occur for PT investors. We can also avoid these types of problems by adding a positive wealth constraint \(W(T) \geq B > 0\) into Problem (PT'), which does not alter our general conclusions.
where $y^{RA} \geq 0$ and $y^{PT} \geq 0$ are Lagrange multipliers for the budget constraints of Problem (RA') and (PT'), respectively; $\zeta$ solves Eq. (12); and

\[
v_y = \left( \psi(y^{RA})^{1/(\gamma-1)} (1 - \psi)(y^{PT})^{1/(\gamma-1)} \right)^{-1},
\]

\[
a(t) = \frac{1}{\eta_1} (e^{\eta_1(T-t)} - 1), \quad \eta_1 = \gamma \mu_D - 1/2 \gamma(1 - \gamma) \sigma_D^2,
\]

\[
b(t) = e^{\eta_2(T-t)}, \quad \eta_2 = \frac{1 - \gamma}{1 - \alpha_2} \alpha_2 \mu_D - \frac{1}{2} \alpha_2 \left( \frac{1 - \frac{1}{2} (1 - \alpha_2)}{1 - \alpha_2} \right) \sigma_D^2,
\]

\[
d_1(D, t) = \frac{\ln \zeta + (1 - \gamma)(\ln D(t) + \ln v_y) + (r_f - \frac{1}{2} \theta^2)(T - t)}{\theta \sqrt{T - t}},
\]

\[
d_2(D, t) = d_1(D, t) + \frac{\theta \sqrt{T - t}}{1 - \alpha_2},
\]

\[
\Gamma(t) = \frac{\alpha_2}{1 - \alpha_2} \left( r_f + \frac{1}{2} \theta^2 \right)(T - t) + \frac{1}{2} \left( \frac{\alpha_2}{1 - \alpha_2} \right)^2 \theta^2(T - t).
\]

This stock price process is no longer a geometric Brownian motion and hence differs from the process in the previous section considerably.

However, Berkelaar and Kouwenberg (2009) do not present the equilibrium stock holdings of CRRA and PT investors in their study because their main objective is to examine the implications of PT preferences on stock price formation. In contrast, our focus is on the analysis of trading behavior, which is directly determined by changes in investors’ stock holdings. The following proposition presents the equilibrium stock holdings and the related return dynamics in the market interaction case.

**Proposition 4.** (i) At time $t$, the equilibrium stock holdings (demands) of the CRRA and PT investors are as follows:

\[
H^{HA}_e(t) = \frac{H_{1,e}(t) + H_{1,w}(D, t)}{H_S(D, t)} , \quad H^{PT}_e(t) = \frac{H_{2,e}(t) + H_{2,w}(D, t)}{H_S(D, t)}.
\]

where

\[
q(t) = \psi + (1 - \psi) \frac{H_{2,e}(t) + H_{2,w}(D, t)}{H_{1,e}(t) + H_{1,w}(D, t)},
\]

18
in the benchmark case of homogeneity, i.e., the case where \( \psi \) emphasizes that preference heterogeneity generates trades; otherwise, there is no motive to trade now scaled by the market-clearing condition as in Eq. (15). As such, the market interaction case serves the forms of (20) illustrates that the equilibrium stock holdings are in sharp contrast to the previous partial-serve the forms of (ii) The excess return process for investing in one share of the stock is given by

\[
dR(t) = (D(t) - r_f S(t)) dt + dS(t) = \mu_R(t) dt + \sigma_R(t) dB(t), \quad R(0) = 0,
\]

and its drift and volatility terms are given by

\[
\mu_R(t) = \theta \sigma_R(t) = q(t) \mu_R^B(t), \quad \sigma_R(t) = \sigma_D D(t) H_S(D(t)) = q(t) \sigma_R^B(t),
\]

where \( \mu_R^B(t) \) and \( \sigma_R^B(t) \) are the drift and volatility terms in the benchmark economy without PT investors (i.e., the case \( \psi = 1 \)).

Proof: See the Appendix.

It is immediately possible to verify that \( H_{2,W}(D, t) \) and \( H_{1,W}(D, t) \) (as functions of \( D(t) \)) preserve the forms of \( H^{\psi}(t) \) and \( H^{\alpha}(t) \) (as functions of \( S(t) \)) discussed in Section 2.\(^7\) However, Eq. (20) illustrates that the equilibrium stock holdings are in sharp contrast to the previous partial-equilibrium results. The reason for this difference is that the stock holdings of different agents are now scaled by the market-clearing condition as in Eq. (15). As such, the market interaction case emphasizes that preference heterogeneity generates trades; otherwise, there is no motive to trade in the benchmark case of homogeneity, i.e., the case where \( \psi = 1 \).

The factor \( q(t) \) then represents a sensitivity multiple caused by preference heterogeneity relative to the benchmark case of homogeneity, a no-trade case. Eq. (22) shows that \( q(t) \) measures the

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\(^7\)This results from the fact that the state price density \( \zeta(t) \) is still a geometric Brownian motion in equilibrium, as in the previous section. (Hence, those optimal wealth solutions with respect to \( \zeta(t) \) remain feasible.)
magnitude of the extra volatility (drift) component induced by preference heterogeneity, i.e.,

\[ q(t) = \frac{\sigma_R(t)}{\sigma^R_R(t)} = \frac{\mu_R(t)}{\mu^R_R(t)}. \]

Moreover, \( q(t) \) is directly related to the stock holdings of PT investors in equilibrium because \( q(t) \) is proportional to the relative demand ratios

\[ \frac{H_{2,c}(t) + H_{2,w}(D, t)}{H_{1,c}(t) + H_{1,w}(D, t)} = \frac{H^*_{PT}(t)}{H^*_{RA}(t)}. \] (23)

Then, differentiating Eqs. (15), (21), and (23), we have

\[ \text{sign} \left( \frac{\partial H^*_{PT}(t)}{\partial S(t)} \right) = \text{sign} \left( \frac{\partial \left[ H^*_{PT}(t) / H^*_{RA}(t) \right]}{\partial S(t)} \right) = \text{sign} \left( \frac{\partial q(t)}{\partial S(t)} \right). \] (24)

That said, if preference heterogeneity leads PT investors to trade in the opposite direction as price changes, it also causes the volatility and drift of stock returns to vary (in addition to the variations in the benchmark no-trade case) in the same contrarian fashion. As such, we make the following remark.

**Remark 5.** The market interaction will reconcile the negative-feedback trading behavior of PT investors with a source of asymmetric volatility (i.e., volatility is negatively correlated with the realized stock returns) and short-run return reversal (i.e., expected future returns are negatively correlated with the realized stock returns).

The intuition is straightforward: market clearing necessarily affects trade behavior through its effect on price behavior. Specifically, the investors’ trading actions and the stock return process are both endogenously and simultaneously determined in equilibrium. As both the equilibrium riskless rate and the market price of risk are constants, the attractiveness of the stock with respect to the bond (for a CRRA investor) is controlled by its return volatility. That said, CRRA investors trade in the opposite direction as the volatility changes unexpectedly. As such, whenever PT investors increase (decrease) their stock holdings, the volatility relative to the benchmark no-trade case will increase (decrease) so as to clear the market, i.e., to induce the CRRA investors to accommodate
the trades of the PT investors. Because the market price of risk remains unchanged, the expected stock premium relative to the benchmark case must change accordingly.

Eq. (24) illustrates that the relative demand ratio in Eq. (23) determines the direction of the PT investors’ response to price changes. As both $H_{1,c}(t)$ and $H_{2,c}(t)$ are only functions of $t$, the sensitivity of this relative demand ratio to price changes (or dividend shocks) is primarily determined by the ratio

$$\delta_w(D, t) = H_{2,W}(D, t)/H_{1,W}(D, t).$$

Because $H_{2,W}(D, t)$ and $H_{1,W}(D, t)$ inherit the functional forms of $H^{PT}(t)$ and $H^{RA}(t)$ discussed in Section 2, the ratio $\delta_w(D, t)$ does indeed have a similar form to the loss aversion component $\delta(t)$ established in Eq. (7) and Proposition 3. This property thus suggests that the loss aversion component can manifest itself in the equilibrium trading behavior of PT investors.

Figure 4 provides a description of the above thinking with the parameters specified as follows. We set $\psi$ at 0.5 and the dividend parameters at $\mu_D = 0.056$ and $\sigma_D = 0.115$ based on monthly S&P500 index data from 1980-1999. When we set the risk aversion parameter in the consumption utility, $\gamma$, at $-1.5$, the implied equilibrium market price of risk is 0.29, which is close to the historical average Sharpe ratio of approximately 0.30. For the remaining parameters, we assume that all investors have initial wealth $W_0 = 1$, a time horizon $T = 1$, and a scaling parameter $\rho = 15$. For the value function of the PT investors, we use the estimates of Tversky and Kahneman (1992) and the reference level $\bar{W} = W_0 e^{-rfT}$.

Figure 4a shows that the relative demand variables $H^{PT}(t)/H^{RA}(t)$ and $\delta_w(t)$ have similar patterns with respect to the dividend growth rate $\ln(D(t)/D(0))$. Figure 4b reveals the anticipated result that

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8 In fact, consumption reasons also appear to be inappropriate for directly explaining anomalous behavioral patterns in financial markets such as short-term contrarian behavior and the disposition effect.

9 The following analysis reveals that our basic results are robust to the values of these parameters. We consider 1980-1999 here because this period contains the period 1987-1993 considered by Odean (1998). Our calibration work in Section 4 is based on his empirical study.
the case $\partial H^{PT}_e(t)/\partial S(t) < 0$ dominates in equilibrium,\textsuperscript{10} although we use the parameters of Tversky and Kahneman (1992) to characterize the wealth preference of PT investors. Indeed, to a large extent, the price sensitivity of stock demand $\partial H^{PT}_e(t)/\partial S(t)$ (as a function of $D(t)$) inherits the non-monotonic form of $\varepsilon^{LA}$ (as a function of $S(t)$) seen in Figure 1. The figure also shows that CRRA investors typically exhibit a positive-feedback trading tendency. The reason is obvious: they must be on the other side of trades. In short, we find the following.

**Remark 6.** In our market populated by heterogeneous investors, PT investors typically exhibit a negative-feedback trading tendency while CRRA investors do the opposite.

Figure 5 further depicts the Iso-lines of negative-feedback-trading probability ($\text{Pr}(\partial H^{PT}_e(t)/\partial S(t) < 0)$) in the $(\alpha_2, \lambda)$ and $(t, \gamma)$ coordinate systems. The figure shows that the negative-feedback-trading probability maintains a high level over a wide range of parameterizations, confirming Remark 6. Figure 5 shows that increasing $\lambda$ or decreasing $t$ leads to a more prominent negative-feedback-trading probability for PT investors. Similar properties can also be found in Figure 2. This consistency confirms our previous argument that incorporating market interaction can lead to a dominance of the loss aversion component in equilibrium behavior.

For the sake of comparison, Figure 6 replicates the situation of Figure 4 and plots $q(t)$ against $\ln(D(t)/D(0))$ and $S(t)$ for various values of $\psi$. Figure 6a illustrates that $q(t)$’s pattern is consistent with the demand pattern presented in Figure 4a. Figure 6b reveals virtually the same patterns as those in Figure 6a because the stock price in equilibrium is a monotonic function of the dividend flow $D(t)$. Figure 6 also shows that $q(t)$ and its slope (e.g., with respect to $S(t)$) increase when the

\textsuperscript{10}The relationship between stock demand and stock price becomes much more complicated in the equilibrium setting. In this section, we approximate the partial derivative with respect to $S(t)$ as follows:

$$\frac{\partial H(t)}{\partial S(t)} \approx \frac{\Delta H(t)}{\Delta S(t)},$$

where $\Delta S(t) = S(t; B(t) + h/2) - S(t; B(t) - h/2)$; there is a similar expression for $\Delta H(t)$. We take $h = 0.01 \sqrt{t}$ throughout and find that this value is small enough to maintain high accuracy in our calculation.
proportion of PT investors \((1 - \psi)\) increases.

As is pointed out by Berkelaar and Kouwenberg (2009), the downward curves in the figures are consistent with the asymmetric volatility effect. In this paper, we emphasize that the downward curves imply the occurrence of short-run return reversals together with negative-feedback trading activities.

The reconciliation of return reversals with negative-feedback trading behavior contrasts with the view of Grinblatt and Han (2005) that PT preferences can lead to momentum in stock returns by generating trading actions that are consistent with the disposition effect. In their interaction model, however, Grinblatt and Han (2005) omit the link between PT and the disposition effect, and take as given the demand functions of “rational” investors, who always try to keep stock prices close to their fundamental values, and the functions of PT investors featured by a disposition effect. As opposed to using this mechanistic trader approach, we endogenize both trade and price behavior by explicitly modeling investor psychology in the form of CRRA and PT preferences. Our analysis further illustrates that the two implementations of PT at the levels of individual choice and market interaction, although closely related, deliver very different stock demand functions with respect to stock price. Indeed, market interaction is crucial in our model for linking PT preferences with negative-feedback trading behavior. As such, the particular type of contrarian behavior examined in Grinblatt and Han’s model is essentially distinct from that derived in this work.

In fact, the endogenously contrarian behavior of PT investors examined here gives rise to “noise trading.” As discussed, the aggregate demand shocks of PT investors become a source of changes in the stock’s riskiness and short-term return reversals. The properties are consistent with those predicted by the models of noise trading such as in De Long et al. (1990); Campbell and Kyle (1993); Campbell et al. (1993). Campbell et al. (1993) further point out that the basic intuition of the noise trading models “carries through regardless of how non-informational trading is introduced.” A contribution of this section is therefore to explicitly work out the implications of PT preferences for deriving non-informational trading.
4. Revisiting the disposition effect based on empirical indicants

The elasticity approach in the previous analysis presumes that we use instantaneous stock prices as reference points when calculating stock gains and losses. In contrast, the literature often defines the reference point underlying the disposition effect as that derived from one’s personal financial history. For example, Odean (1998) uses the average stock purchase price as the reference point. In this section, we test whether some of our conclusions are still valid when we use the method of Barberis and Xiong (2009), who conduct their simulation analysis based directly on the empirical indicants offered by Odean (1998). To address this issue, we provide a simple empirical calibration of PT’s parameters and conduct comparative static exercises of the calibrated results based on the individual-choice model in Section 2.11

We then calibrate the model using aggregate data from the financial literature about the U.S. stock market. For the asset return process, we use the estimates of Kandel and Stambaugh (1996) based on U.S. equity returns from 1927-1993: \( r_f = 0.028 \), \( \sigma = 0.194 \), and \( \theta = 0.303 \).

We access the actual trading activity based on 12 monthly estimates for the individual’s trading patterns obtained by Odean (1998). He obtains these estimates based on the stock-trading behavior of 10,000 households from 1987-1993.12 To capture the disposition effect, Odean (1998) defines two measures, the Proportion of Gains Realized (PGR) and the Proportion of Losses Realized (PLR), as follows:

\[
PGR = \frac{\text{no. of realized gains}}{\text{no. of realized gains} + \text{no. of paper gains}},
\]

\[
PLR = \frac{\text{no. of realized losses}}{\text{no. of realized losses} + \text{no. of paper losses}},
\]

where

11 As the equilibrium model in Section 3 cannot be easily tested with aggregate data (Campbell, 2000), we do not extend the calibration to the market interaction case.
12 We are very grateful for Odean’s generous support in providing us with these estimates.
• a “realized gain (loss)” means that a stock is sold and its price exceeds (does not exceed) the average price at which the shares were purchased;

• a “paper gain (loss)” means that a stock is not sold and its price exceeds (does not exceed) the average price at which the shares were purchased.

In other words, PGR measures the tendency of investors to realize their gains, and PLR measures the tendency of investors to realize their losses. If the ratio of PGR to PLR is bigger than 1, which is conceptually consistent with the condition in (6), we observe a disposition effect. Based on the dataset, Odean (1998) calculates the ratio for each month (see, e.g., Figure 7), which decreases from 2.11 in January to 0.85 in December.

For the investor’s subjective parameters, we try to stay consistent with the standard literature. As is common in many studies about PT, we set $T = 1$ (one year) and $W = W_0 e^{rT}$ in the calibration.\(^\text{13}\) As mentioned previously, we also fix $\alpha_1 = 0.88$ in this section.

To link PT preferences with the actual trading patterns, we then calculate the ratio PGR/PLR of a PT investor following the simulation method of Barberis and Xiong (2009). In addressing this issue, we adopt the usual numerical procedures to approximate our continuous-time model on a discrete grid. Specifically, we divide a year into $n = 12 \times 8 = 96$ trading periods of size $\Delta t = 1/n$. Accordingly, the stock price process is discretized as follows:

$$S_{t+\Delta t} - S_t = S_t(\mu \Delta t + \sigma z_t),$$

where $z$ is a pseudo-random number drawn from a normal distribution with a mean of 0 and a variance of $\Delta t$. Through this equation, we then generate 100,000 stock price paths. For every path, given parameters $(\theta, r_f, \sigma, T, \bar{W}, \lambda, \alpha_1, \alpha_2)$, we can compute the investor’s stock demand $H(t)$

\(^{13}\)As Benartzi and Thaler (1995) point out, “we file taxes once a year and receive our most comprehensive mutual fund reports once a year; moreover, institutional investors scrutinize their money managers’ performance most carefully on an annual basis.”
in various cases and then count the corresponding numbers for “realized gain,” “realized loss,” “paper gain,” and “paper loss.” Based on these numbers, we can then obtain the PT investor’s ratios of PGR to PLR for every month, as did Odean (1998). The monthly ratios further allow us to examine the link between PT and the disposition effect on the seasonality dimension, which is not addressed by Barberis and Xiong (2009).

The calibration question now becomes: what combination of $\alpha_2$ and $\lambda$ can offer the best fit to the phenomenon observed by Odean (1998)? We use the least squares criterion to answer this question, i.e.,

$$\min_{\alpha_2, \lambda} f(\alpha_2, \lambda) = \sum_{i=1}^{12} (\hat{R}_i(\alpha_2, \lambda) - R_i)^2,$$

where $\hat{R}(\alpha_2, \lambda)$ denotes the ratios of PGR to PLR obtained from the portfolio model (PT), and $R$ denotes the estimates obtained by Odean (1998). We use the idea of the finite-difference method to solve the problem. Specifically, we calculate $f(\alpha_2, \lambda)$ for the grids $\{\alpha_2, \lambda\}$ ($i = 1, \ldots, n$ and $j = 1, \ldots, m$) in the form of an $n \times m$ matrix $F$, and report the combination $(\alpha_2, \lambda_j)$ that corresponds to the minimum value of $F$.

Figure 7 compares the values of the PGR/PLR ratio obtained by Odean (1998) to those generated by the calibrated results. The figure shows that the pair of parameters,

$$\alpha_2 = 0.74 \text{ and } \lambda = 1.61,$$

offers a rather good fit to the estimates of Odean (1998). In contrast, the figure also shows that the standard implementation of PT suggested by Tversky and Kahneman (1992) (i.e., $\lambda = 2.25$, $\alpha_1 = \alpha_2 = 0.88$) turns out to be problematic for generating a salient disposition effect. Consistent with Remark 4, our fit value of $\alpha_2$ is lower than 0.88.

Figure 8 further displays some comparative statics and shows how the changes in the calibrated parameters alter the ratios for the disposition effect. Figure 8a shows that the PGR/PLR ratios

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14Here, we define the mesh size of the grid as $\alpha_2, i - \alpha_2, i-1 = 0.01$ and $\lambda_j - \lambda, j-1 = 0.01$. 

26
decrease as $\alpha_2$ increases. Figure 8b shows that decreasing $\lambda$ tends to decrease the PGR/PLR ratios, although its effect is rather unpronounced. The results are consistent with the patterns displayed in Figure 3.

To sum up, the simulation results in this section are in accordance with those theoretical results reported previously, despite the use of different definitions and measurements. The compatibility is consistent with the findings of Odean (1998) and some other studies that the disposition effect is robust to various reference point choices in calculating stock gains and losses, such as the average purchase price, the highest purchase price, the first purchase price, and the most recent purchase price. In turn, the compatibility also confirms that the elasticity approach used in this paper can be considered a useful technique in investigating those empirical facts about trading activity in theoretical models.

5. Conclusion

Our study analyzes how PT’s value function gives rise to preference-based trading patterns that are not captured by conventional concave utility functions defined over wealth. We first disentangle the value function into risk and loss aversion components and find that the loss (risk) aversion component gives rise to negative (positive) feedback trading propensities. As the loss aversion component is manifested as a combination of reference dependence, loss aversion, and risk seeking for losses, we can still think of these distinctive psychological concepts introduced by PT as the behavioral causes of negative-feedback trading.15

15In fact, there are also empirical studies that clearly attribute negative-feedback trading propensities to those unconventional features of the value function, rather than vaguely taking it as the sole carrier of contrarian trading motives. In a recent empirical study for the Chicago Board of Trade, for example, Coval and Shumway (2005) take reference dependence, loss aversion, and risk seeking for losses as the behavioral motivations for their investigation of a negative relation between profits and subsequent risk-taking. They find evidence that traders in T-bond futures exhibit clearly loss-averse behavior and assume significantly greater afternoon risk following morning losses, and that the traders’ be-
We then extend PT’s trading pattern analysis to a market-interaction level. It turns out that by virtue of the market interaction between PT investors and CRRA investors, the loss aversion component manifests itself, as a basic source of non-informational trading, in equilibrium behavior. In this manner, the interaction reestablishes the intuitive link between PT and negative-feedback trading patterns, and also reconciles this link with asymmetric volatility and short-term return reversals. The properties suggest that PT preferences can lead investors to behave endogenously as contrarian noise traders in the market interaction process.

As we use the basic settings to focus on the implications of PT preferences for negative-feedback trading propensities in this study, there remain some possible directions to explore. For example, in examining various experimental markets, Bloomfield et al. (2009) find that traders who lack informational advantages tend to behave as irrational contrarian noise traders. Within a continuous-time setting, Yao and Li (2013) also demonstrate that, for bounded rational agents, decision-making with incomplete information could naturally occur in a way that reflects loss aversion. Extending our analysis to examine both informational and psychological motives for negative-feedback trading behavior may be worth pursuing, as individual investors are usually observed to be prone to both psychological biases and informational disadvantages.

Appendix

We use the martingale representation approach (e.g., Cox and Huang, 1989) to solve the portfolio choice problems.

Proof of Proposition 1: In the setting of this paper, the optimal wealth at time $t$ satisfies

$$W(t) = \frac{1}{\zeta(t)} \mathbb{E}_t [\zeta(T)W(T)],$$  \hspace{1cm} (A.1)

behavior has important short-term consequences in terms of price change reversals. Clearly, their behavioral hypothesis and empirical results are consistent with our theoretical analysis.
where \( \ln \zeta(T) \) is normally distributed with mean \( \ln \zeta(t) - (r + \frac{\theta^2}{2})(T - t) \) and variance \( \theta^2(T - t) \). Applying the Itô lemma to Eq. (3) yields

\[
H(t) = \frac{\partial W(t)}{\partial S(t)} = \frac{\partial W(t)}{\partial S(t)}/\partial \zeta(t).
\]  

(A.2)

As is well known in the literature, the optimal terminal wealth of the CRRA investor is:

\[
W^{RA}(T) = \left( \frac{y^{RA} \zeta(T)}{\alpha^2} \right)^{1/(\alpha_2 - 1)} ,
\]

where \( y^{RA} > 0 \) satisfies the budget constraint \( E[\zeta(T)W(T)] = \zeta_0 W_0 \).

Substituting the optimal terminal wealth, \( W^{RA}(T) \), into (A.1) yields the optimal wealth process \( W^{RA}(t) \). Differentiating \( W^{RA}(t) \) with respect to \( \zeta(t) \) gives the following

\[
\frac{\partial W^{RA}(t)}{\partial \zeta(t)} = -\frac{1}{1 - \alpha_2} \left( \frac{y^{RA}}{\alpha^2} \right)^{1/(\alpha_2 - 1)} e^{\Gamma(t) \zeta(t) \frac{1}{\alpha - 1} - 1}.
\]  

(A.3)

Within the Black-Scholes market considered in this work, we can obtain the one-to-one relationship between \( \zeta(t) \) and \( S(t) \) as follows:

\[
S(t) = S_0 \left[ \frac{\zeta_0}{\zeta(t)} \left( \frac{\zeta(t)}{\zeta_0} \right)^{1/(\alpha_2 - 1)} \right]^{\sigma/\theta}.
\]  

(A.4)

Differentiating this equation with respect to \( \zeta(t) \) then yields

\[
\frac{\partial S(t)}{\partial \zeta(t)} = -\frac{\sigma \, S(t)}{\theta \, \zeta(t)}.
\]  

(A.5)

Rearranging Eq. (A.4) yields

\[
\zeta(t) = \zeta_0 \left( \frac{S_0}{S(t)} \right)^{\theta/\sigma} e^{\left( \frac{\zeta(t)}{\zeta_0} \right)^{1/(\alpha_2 - 1)}}.
\]

Using this equation to replace \( \zeta(t) \) in Eqs. (A.3) and (A.5), we have \( H^{RA}(t) \) based on Eq. (A.2). Differentiating \( \ln H^{RA}(t) \) with \( \ln S(t) \) further yields \( \epsilon^{RA}(t) \). Q.E.D.

**Proof of Proposition 3:** Berkelaar et al. (2004) have derived the expressions for the optimal wealth process \( W^{PT}(t) \) and the related optimal portfolio weights as functions of \( \zeta(t) \) (see Proposition 6 in their study). For the purpose of this proposition, we need further establish the expression of the
investor’s stock demand $H(t)$, defined by the number of shares, as a function of $S(t)$. Differentiating $W^{PT}(t)$ with respect to $\zeta(t)$ gives

$$
\frac{\partial W^{PT}(t)}{\partial \zeta(t)} = -\frac{\phi(d_1(\zeta, t))}{\theta \sqrt{T-t}} W e^{-r(T-t)\zeta(t)}^{-1}
- \left[ \Phi(d_2(\zeta, t)) + \frac{\phi(d_2(\zeta, t))}{\theta \sqrt{T-t}} \right] (\alpha_2)^{1/(1-\alpha_2)} e^{\Gamma(t)\zeta(t)} - \frac{1}{1-\alpha_2}^{-1},
$$

where

$$
d_1(x, t) = \ln x - \ln(\zeta(t)) + (r_f - \frac{1}{2} \theta^2)(T-t) \theta \sqrt{T-t},
d_2(x, t) = d_1(x, t) + \frac{\theta \sqrt{T-t}}{1-\alpha_2}.
$$

Then, following exactly the same steps as in the proof of Proposition 1 (by replacing $\partial W^{RA}(t)/\partial \zeta(t)$ with $\partial W^{PT}(t)/\partial \zeta(t)$), we obtain the expression for $H^{PT}(t)$ as a function of $S(t)$ and $\overline{S}$. This completes the proof for item (i) of the proposition.

Dividing $H^{PT}(t)$ by $H^{RA}(t)$, together with the fact that the parameter $\alpha_2$ in $H^{RA}(t)$ is kept the same as in $H^{PT}(t)$, yields the ratio $\delta(t)$. Item (ii) of the proposition then follows. Differentiating $\ln \delta(t)$ with respect to $\ln S(t)$, we can derive $\varepsilon^{LA}(t)$ given in item (iii) of the proposition, after some long but straightforward calculations.

Eq. (A.4) shows that $S$ is a strictly decreasing function of $\overline{S}$. Berkelaar et al. (2004) have demonstrated that $\overline{S}$ increases with $\lambda$ and $W_0$ (see Proposition 4 in their study). It is thus straightforward to show that $\overline{S}$ decreases with $\lambda$ and $W_0$.

On the other hand, defining $z = y^{PT}\overline{\zeta}$, we can rewrite Eq. (12) as

$$
g(z, \overline{W}) = 1 - \frac{\alpha_2}{\alpha_2} \left( \frac{1}{z} \right)^{\alpha_2/(1-\alpha_2)} (\alpha_2)^{1/(1-\alpha_2)} - \overline{W}z + \lambda \overline{W}^{\alpha_1} = 0.
$$

Using this equation, we have

$$
\frac{dz}{d\overline{W}} = -\frac{g'(z, \overline{W})}{g'(z, \overline{W})} = \frac{1 - \alpha_2}{\alpha_2} \left( \frac{1}{z} \right)^{\alpha_2/(1-\alpha_2)} (\alpha_2)^{1/(1-\alpha_2)} + \frac{\overline{W}}{\lambda \alpha_1 \overline{W}^{\alpha_1} - \overline{W}z},
$$

and

$$
\overline{W}z = \lambda \overline{W}^{\alpha_1} + \frac{1 - \alpha_2}{\alpha_2} \left( \frac{1}{z} \right)^{\alpha_2/(1-\alpha_2)} (\alpha_2)^{1/(1-\alpha_2)} > \lambda \alpha_1 \overline{W}^{\alpha_1}.
$$
So we have \( d\zeta/dW < 0 \). Then, if we increase \( W_1 \) to \( W_2 \), we know \( z_1 = y_1^{PT} \zeta_1 > z_2 = y_2^{PT} \zeta_2 \).

For the optimal terminal wealth (11), Berkelaar et al. (2004) have demonstrated that the constraint 
\[ E[\zeta(T)W^{PT}(T)] = \zeta_0 W_0 \text{ cannot be satisfied in the case } y_1^{PT} > y_2^{PT} \text{ and } \zeta_1 < \zeta_2, \text{ or in the case } y_1^{PT} < y_2^{PT} \text{ and } \zeta_1 > \zeta_2. \]

Therefore, the only feasible change in the parameters is \( y_1^{PT} > y_2^{PT} \) and \( \zeta_1 > \zeta_2 \). It is then straightforward to obtain \( \bar{S}_1 < \bar{S}_2 \). Item (iv) in Proposition 3 follows. Q.E.D.

Proposition 3 shows that a positive \( H^{PT}(t) \) is automatically achieved in partial equilibrium when investors have PT preferences. This result is obtained because the optimal terminal wealth \( W^{PT}(T) \) is monotonically decreasing in \( \zeta(T) \). Specifically, using Eqs. (11) and (A.1), we have 
\[ \partial W(t)/\partial \zeta(t) < 0. \]
Eq. (A.5) indicates \( \partial S(t)/\partial \zeta(t) < 0. \) Eq. (A.2) then suggests \( H^{PT}(t) > 0, \) i.e., PT preference prevents the short sale of the stock, even though we do not have an explicit short sale constraint. The result is consistent with the empirical evidence (e.g., Barber and Odean, 2008) that short sales seldom occur in the trading activities of individual investors. The same logic applies in the case of \( H^{RA}(t) \) in Proposition 1 and in the cases of \( H^{e}_{RA}(t) \) and \( H^{e}_{PT}(t) \) in Proposition 4.

**Proof of Proposition 4:** For details on how to obtain the wealth processes in equilibrium, we refer to Berkelaar and Kouwenberg (2009). From their solutions, we can obtain the equilibrium wealth processes in this paper as follows:

\[
W^{RA}(t) = a(t)\left(\frac{\gamma^{RA}}{\rho_1}\right)^{1/(\gamma-1)}v_y D(t) + b(t) \left(\frac{\gamma^{RA}}{\rho_1}\right)^{1/(\alpha_1-1)} (v_y D(t))^{(1-\gamma)/(1-\alpha_2)} ,
\]
\[
W^{PT}(t) = a(t)\left(\frac{\gamma^{PT}}{\rho_1}\right)^{1/(\gamma-1)}v_y D(t) + \bar{W} e^{-r(T-t)}\Phi(d_1(D,t))
+ \left(\frac{\rho_2 \alpha_2}{\gamma^{PT}}\right)^{1/(1-\alpha_2)} e^{\Gamma(t)}(v_y D(t))^{(1-\gamma)/(1-\alpha_2)} \Phi(d_2(D,t)).
\]

At the same time, the equilibrium conditions require
\[
S(t) = \psi W^{RA}(t) + (1 - \psi) W^{PT}(t).
\]

Again, using the Itô lemma, we have
\[
H(t) = \frac{\partial W(t)}{\partial S(t)} = \frac{\partial W(t)/\partial D(t)}{\partial S(t)/\partial D(t)}.
\]
Differentiating $W(t)$ and $S(t)$, we obtain

\[
\frac{\partial W^{RA}(t)}{\partial D(t)} = H_{1,c}(t) + H_{1,w}(D, t) > 0,
\]
\[
\frac{\partial W^{PT}(t)}{\partial D(t)} = H_{2,c}(t) + H_{2,w}(D, t) > 0,
\]
\[
\frac{\partial S(t)}{\partial D(t)} = \psi(H_{1,c}(t) + H_{1,w}(D, t)) + (1 - \psi)(H_{2,c}(t) + H_{2,w}(D, t)) > 0.
\]

$H^{RA}_c(t)$ and $H^{PT}_c(t)$ in item (i) then follow.

The owner of a stock enjoys two sources of income: the capital gain and the dividend. The sum of the two defines the return of the stock, which is given by $D(t)dt + dS(t)$. The excess return process then follows by comparing the stock return with the riskless return. Q.E.D.

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Figure 1: Elasticity $\varepsilon^{\lambda}(t)$ as a function of $S(t)$. (The default parameters are $r_f = 0.028$, $\sigma = 0.194$, $\theta = 0.303$, $\lambda = 2.25$, $\alpha_1 = \alpha_2 = 0.88$, $T = 1$, $W_0 = 1$, $\overline{W} = W_0 e^{r_f T} \approx 1.03$, $S_0 = 1$, and $t = 0.5$.)
(a) The effect of $\lambda$ ($\alpha_2 = 0.88$)

(b) The effect of $\alpha_2$ ($\lambda = 2.25$)

Figure 2: Probability of the case $e^{\lambda}(t) < 0$. (The default parameters are $r_f = 0.028$, $\sigma = 0.194$, $\theta = 0.303$, $\alpha_1 = 0.88$, $T = 1$, $W_0 = 1$, $\bar{W} = W_0 e^{r_f T}$, $S_0 = 1$, and $t = 0.5$.)

Figure 3: Iso negative-elasticity-probability ($\Pr(e^{\alpha_2}(t) < 0)$) lines in the $(\alpha_2, \lambda)$ plane. (The default parameters are $r_f = 0.028$, $\sigma = 0.194$, $\theta = 0.303$, $\alpha_1 = 0.88$, $t = 0.5$, $T = 1$, $W_0 = 1$, $\bar{W} = W_0 e^{r_f T}$, and $S_0 = 1$.)
Figure 4: The equilibrium stock demand and trading pattern of PT investors. (The default parameters are $\mu_D = 5.6\%$, $\sigma_D = 11.5\%$, $\psi = 0.5$, $\rho_1 = \rho_2 = 15$, $\gamma = -1.5$, $\alpha_1 = \alpha_2 = 0.88$, $\lambda = 2.25$, $T = 1$, $W_0 = 1$, $\bar{W} = W_0 e^{-r_f T}$, and $t = 0.5$.)

Figure 5: Iso negative-feedback-trading probability ($\text{Pr}(\partial H^T_e(t) / \partial S(t) < 0)$) lines. (The default parameters are $\mu_D = 5.6\%$, $\sigma_D = 11.5\%$, $\psi = 0.5$, $\rho_1 = \rho_2 = 15$, $\gamma = -1.5$, $\alpha_1 = \alpha_2 = 0.88$, $\lambda = 2.25$, $T = 1$, $W_0 = 1$, $\bar{W} = W_0 e^{-r_f T}$, and $t = 0.5$.)
Figure 6: Magnitude of the extra volatility (premium) component attributed to the presence of PT investors where \( q(t) = \frac{\sigma_R(t)}{\sigma_R^B(t)} = \frac{\mu_R(t)}{\mu_R^B(t)} \). (The default parameters are the same as those in Figure 4.)

Figure 7: Monthly ratios of PGR to PLR. (The default parameters are \( r_f = 0.028, \sigma = 0.194, \theta = 0.303, \alpha_1 = 0.88, T = 1, W_0 = 1, \overline{W} = W_0 e^{r_f T}, \) and \( S_0 = 1 \).)
Figure 8: PGR/PLR ratios with respect to $\alpha_2$ and $\lambda$. (The default parameters are $r_f = 0.028$, $\sigma = 0.194$, $\theta = 0.303$, $\alpha_1 = 0.88$, $T = 1$, $W_0 = 1$, $\bar{W} = W_0 e^{r_f T}$, $S_0 = 1$, $\alpha_2 = 0.74$, and $\lambda = 1.61$.)