Chapter 9
Basic Oligopoly Models
Overview

I. Conditions for Oligopoly?
II. Role of Strategic Interdependence
III. Profit Maximization in Four Oligopoly Settings
   – Sweezy (Kinked-Demand) Model
   – Cournot Model
   – Stackelberg Model
   – Bertrand Model
IV. Contestable Markets
Oligopoly Environment

- Relatively few firms, usually less than 10.
  - Duopoly - two firms
  - Triopoly - three firms
- The products firms offer can be either differentiated or homogeneous.
- Firms’ decisions impact one another.
- Many different strategic variables are modeled:
  - No single oligopoly model.
Role of Strategic Interaction

- Your actions affect the profits of your rivals.
- Your rivals’ actions affect your profits.
- How will rivals respond to your actions?
An Example

- You and another firm sell differentiated products.

- How does the quantity demanded for your product change when you change your price?
D₂ (Rival matches your price change)

D₁ (Rival holds its price constant)
Demand if Rivals Match Price Reductions but not Price Increases

D_2 (Rival matches your price change)

D_1 (Rival holds its price constant)

P

Q

Q_0

P_0
Key Insight

- The effect of a price reduction on the quantity demanded of your product depends upon whether your rivals respond by cutting their prices too!
- The effect of a price increase on the quantity demanded of your product depends upon whether your rivals respond by raising their prices too!
- Strategic interdependence: You aren’t in complete control of your own destiny!
Sweezy (Kinked-Demand) Model
Environment

- Few firms in the market serving many consumers.
- Firms produce differentiated products.
- Barriers to entry.
- Each firm believes rivals will match (or follow) price reductions, but won’t match (or follow) price increases.
- Key feature of Sweezy Model
  - Price-Rigidity.
Sweezy Demand and Marginal Revenue

\[ \begin{align*}
\text{D}_2 & \text{ (Rival matches your price change)} \\
\text{D}_S & \text{: Sweezy Demand} \\
\text{MR}_S & \text{: Sweezy MR}
\end{align*} \]
Sweezy Profit-Maximizing Decision

- **D₂ (Rival matches your price change)**
- **D₁ (Rival holds price constant)**
- **Dₜ : Sweezy Demand**
- **P₀**
- **MC₁, MC₂, MC₃**
- **MRₗ**
Sweezy Oligopoly Summary

- Firms believe rivals match price cuts, but not price increases.
- Firms operating in a Sweezy oligopoly maximize profit by producing where
  \[ \text{MR}_S = \text{MC}. \]
  - The kinked-shaped marginal revenue curve implies that there exists a range over which changes in MC will not impact the profit-maximizing level of output.
  - Therefore, the firm may have no incentive to change price provided that marginal cost remains in a given range.
Cournot Model Environment

- A few firms produce goods that are either perfect substitutes (homogeneous) or imperfect substitutes (differentiated).
- Firms’ control variable is output in contrast to price.
- Each firm believes their rivals will hold output constant if it changes its own output (The output of rivals is viewed as given or “fixed”).
- Barriers to entry exist.
Inverse Demand in a Cournot Duopoly

- Market demand in a homogeneous-product Cournot duopoly is

\[ P = a - b(Q_1 + Q_2) \]

- Thus, each firm’s marginal revenue depends on the output produced by the other firm. More formally,

\[ MR_1 = a - bQ_2 - 2bQ_1 \]

\[ MR_2 = a - bQ_1 - 2bQ_2 \]
Best-Response Function

- Since a firm’s marginal revenue in a homogeneous Cournot oligopoly depends on both its output and its rivals, each firm needs a way to “respond” to rival’s output decisions.
- Firm 1’s best-response (or reaction) function is a schedule summarizing the amount of $Q_1$ firm 1 should produce in order to maximize its profits for each quantity of $Q_2$ produced by firm 2.
- Since the products are substitutes, an increase in firm 2’s output leads to a decrease in the profit-maximizing amount of firm 1’s product.
Best-Response Function for a Cournot Duopoly

- To find a firm’s best-response function, equate its marginal revenue to marginal cost and solve for its output as a function of its rival’s output.

- Firm 1’s best-response function is ($c_1$ is firm 1’s MC)
  \[ Q_1 = r_1(Q_2) = \frac{a - c_1}{2b} - \frac{1}{2} Q_2 \]

- Firm 2’s best-response function is ($c_2$ is firm 2’s MC)
  \[ Q_2 = r_2(Q_1) = \frac{a - c_2}{2b} - \frac{1}{2} Q_1 \]
Graph of Firm 1’s Best-Response Function

\[ Q_1 = r_1(Q_2) = \frac{a-c_1}{2b} - 0.5Q_2 \]

(Firm 1’s Reaction Function)
Cournot Equilibrium

- Situation where each firm produces the output that maximizes its profits, given the output of rival firms.
- No firm can gain by unilaterally changing its own output to improve its profit.
  - A point where the two firm’s best-response functions intersect.
Graph of Cournot Equilibrium

\[ Q_1^* \quad \text{and} \quad Q_2^* \]

Cournot Equilibrium

\[ \frac{(a-c_1)}{b} \quad \text{and} \quad \frac{(a-c_2)}{b} \]
Summary of Cournot Equilibrium

- The output $Q_1^*$ maximizes firm 1’s profits, given that firm 2 produces $Q_2^*$.
- The output $Q_2^*$ maximizes firm 2’s profits, given that firm 1 produces $Q_1^*$.
- Neither firm has an incentive to change its output, given the output of the rival.
- Beliefs are consistent:
  - In equilibrium, each firm “thinks” rivals will stick to their current output – and they do!
Firm 1’s Isoprofit Curve

- The combinations of outputs of the two firms that yield firm 1 the same level of profit

\[ \pi_1 = $100 \]

\[ \pi_1 = $200 \]
Another Look at Cournot Decisions

Firm 1’s best response to $Q_2^*$

$\pi_1 = $100

$\pi_1 = $200
Another Look at Cournot Equilibrium
Impact of Rising Costs on the Cournot Equilibrium

Cournot equilibrium after firm 1’s marginal cost increase

Cournot equilibrium prior to firm 1’s marginal cost increase
Collusion Incentives in Cournot Oligopoly
Stackelberg Model Environment

- Few firms serving many consumers.
- Firms produce differentiated or homogeneous products.
- Barriers to entry.
- Firm one is the leader.
  - The leader commits to an output before all other firms.
- Remaining firms are followers.
  - They choose their outputs so as to maximize profits, given the leader’s output.
Stackelberg Equilibrium

Follower's Profits Decline

Stackelberg Equilibrium

\( Q_1 \)

\( Q_2 \)

\( \pi_1^C \)

\( \pi_2^C \)

\( \pi_F^S \)

\( r_1 \)

\( r_2 \)

\( Q_1^C \)

\( Q_1^S \)

\( Q_1^M \)

\( Q_2^C \)

\( Q_2^S \)
The Algebra of the Stackelberg Model

- Since the follower reacts to the leader’s output, the follower’s output is determined by its reaction function
  \[ Q_2 = r_2(Q_1) = \frac{a - c_2}{2b} - 0.5Q_1 \]

- The Stackelberg leader uses this reaction function to determine its profit maximizing output level, which simplifies to
  \[ Q_1 = \frac{a + c_2 - 2c_1}{2b} \]
Stackelberg Summary

- Stackelberg model illustrates how commitment can enhance profits in strategic environments.
- Leader produces *more* than the Cournot equilibrium output.
  - Larger market share, higher profits.
  - First-mover advantage.
- Follower produces *less* than the Cournot equilibrium output.
  - Smaller market share, lower profits.
Bertrand Model Environment

- Few firms that sell to many consumers.
- Firms produce identical products at constant marginal cost.
- Each firm independently sets its price in order to maximize profits (price is each firms’ control variable).
- Barriers to entry exist.
- Consumers enjoy
  - Perfect information.
  - Zero transaction costs.
Bertrand Equilibrium

- Firms set $P_1 = P_2 = MC!$ Why?
- Suppose $MC < P_1 < P_2$.
- Firm 1 earns $(P_1 - MC)$ on each unit sold, while firm 2 earns nothing.
- Firm 2 has an incentive to slightly undercut firm 1’s price to capture the entire market.
- Firm 1 then has an incentive to undercut firm 2’s price. This undercutting continues...
- Equilibrium: Each firm charges $P_1 = P_2 = MC$. 
Contestable Markets

§ Key Assumptions
- Producers have access to same technology.
- Consumers respond quickly to price changes.
- Existing firms cannot respond quickly to entry by lowering price.
- Absence of sunk costs.

§ Key Implications
- Threat of entry disciplines firms already in the market.
- Incumbents have no market power, even if there is only a single incumbent (a monopolist).
A Synthesizing Example

Suppose $P = 100 - Q$ is the market demand

For any firm in this market, $MC = $10, $ATC = $10

Calculate the market output, market price and individual firm profit.

- If the market structure is perfect competition
- If the market structure is monopoly
- If the market structure is Cournot duopoly
- If the market structure is Stackelberg duopoly
- If the market structure is Bertrand duopoly
Perfect Competition

Suppose $P = 100 - Q$ is the market demand.
For any firm in this market, $MC = $10, $ATC = $10.

- $P = MR = MC = 10$ (market price)
- $P = 100 - Q = 10$
- $Q_{PC} = 90$ (market output)
- $P_{PC} = 100 - 10 = 90$
- Profit = $(P_{PC} - ATC) * Q_{PC} = (10 - 10) * 90 = 0$
Monopoly

Suppose $P = 100 - Q$ is the market demand.
For any firm in this market, $MC = $10, $ATC = $10.

- $MR = 100 - 2Q = MC = 10$
- $2Q = 90 \Rightarrow Q_M = 45$ (market output)
- $P_M = 100 - Q_M = 100 - 45 = 55$ (market price)
- $\text{Profit} = (P_M - ATC) \times Q_M = (55 - 10) \times 45 = $2025
Cournot Duopoly

Suppose $P = 100 - Q$ is the market demand where $Q = q_1 + q_2$ ($q_1$ firm 1’s output, $q_2$ is firm 2’s output).

For any firm in this market, $MC = $10, $ATC = $10.

- Firm 1: $P = (100 - q_2) - q_1$
  
  $TR = P \times q_1$, So $MR = (100 - q_2) - 2q_1$
  
  $MR = MC$, so $(100 - q_2) - 2q_1 = 10$
  
  $2q_1 = 90 - q_2$
  
  $q_1 = 45 - 0.5q_2 \Rightarrow$ firm 1’s reaction function
Cournot Duopoly

- Firm 2: \( P = (100 - q_1) - q_2 \)
  
  \[
  TR = P \times q_2, \text{ So } MR = (100 - q_1) - 2q_2
  \]
  
  \[
  MR = MC, \text{ so } (100 - q_1) - 2q_2 = 10
  \]
  
  \[2q_2 = 90 - q_1\]
  
  \[q_2 = 45 - 0.5q_1 \Rightarrow \text{ firm 2’s reaction function}\]

- Plug firm 2’s reaction function into firm 1’s.

  \[
  q_1 = 45 - 0.5q_2 = 45 - 0.5(45 - 0.5q_1)
  \]
  
  \[
  q_1 = 45 - 22.5 + 0.25q_1
  \]
  
  \[0.75q_1 = 22.5 \Rightarrow q_1 = 30 \text{ (firm 1’s output)}\]
  
  \[q_2 = 45 - 0.5 \times 30 = 30 \text{ (firm 2’s output)}\]
Cournot Duopoly

- Market output: $Q_{\text{Cournot}} = q_1 + q_2 = 30 + 30 = 60$
- Market price: $P_{\text{Cournot}} = 100 - Q = 100 - 60 = $40$
- Individual firm’s profit:
  
  Firm 1: $\text{Profit}_1 = (P_{\text{Cournot}} - \text{ATC}) \times q_1$
  
  $= (40 - 10) \times 30 = $900$

  Firm 2: $\text{Profit}_2 = (P_{\text{Cournot}} - \text{ATC}) \times q_2$
  
  $= (40 - 10) \times 30 = $900$

- Total profit: $1800$
Stackelberg Duopoly

Suppose $P = 100 - Q$ is the market demand where $Q = q_1 + q_2$ ($q_1$ firm 1’s output, $q_2$ is firm 2’s output).
For any firm in this market, $MC = $10, $ATC = $10.
Firm 1 is the leader.

- **Follower:** $P = (100 - q_1) - q_2$

  (Firm 2) $TR = P \times q_2$, So $MR = (100 - q_1) - 2q_2$
  $MR = MC$, so $(100 - q_1) - 2q_2 = 10$
  $2q_2 = 90 - q_1$
  $q_2 = 45 - 0.5q_1$ $\Rightarrow$ firm 2’s reaction function
Stackelberg Duopoly

- Leader (Firm 1): \[ P = (100 - q_2) - q_1 \]
  Plug firm 2’s reaction function into firm 1’s residual demand.
  \[ P = [100 - (45 - 0.5q_1)] - q_1 = 55 - 0.5q_1 \]
  \[ TR = P \times q_1, \text{ So } MR = 55 - q_1 \]
  \[ MR = MC, \text{ so } 55 - q_1 = 10 \]
  \[ q_1 = 45 \text{ (firm 1’s output)} \]
- Follower’s output:
  \[ q_2 = 45 - 0.5 \times 45 = 22.5 \text{ (firm 2’s output)} \]
Stackelberg Duopoly

- Market output: \( Q_{\text{Sta}} = q_1 + q_2 = 45 + 22.5 = 67.5 \)
- Market price: \( P_{\text{Sta}} = 100 - Q = 100 - 67.5 = $32.5 \)
- Individual firm’s profit:
  - Firm 1: Profit_1 = \((P_{\text{Sta}} - ATC) \times q_1\)
    \[= (32.5 - 10) \times 45 = $1012.5\]
  - Firm 2: Profit_2 = \((P_{\text{Sta}} - ATC) \times q_2\)
    \[= (32.5 - 10) \times 22.5 = $506.25\]
- Total profit: $1518.75
Betrand Duopoly

Suppose $P = 100 - Q$ is the market demand where $Q = q_1 + q_2$ ($q_1$ firm 1’s output, $q_2$ is firm 2’s output).

For any firm in this market, $MC = $10, $ATC = $10.

- Market price: $P_{\text{Betrand}} = MC = $10
- Market output: $Q_{\text{Betrand}} = 100 - Q = 100 - 10 = 90
- Individual firm’s profit: $0
Example Summary

- **Market price:**
  \[ P_{\text{B}} < P_{\text{S}} < P_{\text{C}} < P_{\text{M}} \]

- **Market output:**
  \[ Q_{\text{B}} > Q_{\text{S}} > Q_{\text{C}} > Q_{\text{M}} \]

- **Individual firm's profit:**
  Firm 1: \[ \pi_{\text{B}} = \pi_{\text{P}} < \pi_{\text{C}} < \pi_{\text{S}} < \pi_{\text{M}} \]
  Firm 2: \[ \pi_{\text{B}} = \pi_{\text{P}} < \pi_{\text{C}} < \pi_{\text{S}} \]
Conclusion

- Different oligopoly scenarios give rise to different optimal strategies and different outcomes.
- Your optimal price and output depends on …
  - Beliefs about the reactions of rivals.
  - Your choice variable (P or Q) and the nature of the product market (differentiated or homogeneous products).
  - Your ability to credibly commit prior to your rivals.