Modelling the optical properties of lossless multilayered spheres

This article has been downloaded from IOPscience. Please scroll down to see the full text article.
2011 J. Opt. 13 095704
(http://iopscience.iop.org/2040-8986/13/9/095704)

View the table of contents for this issue, or go to the journal homepage for more

Download details:
IP Address: 124.160.106.12
The article was downloaded on 19/08/2011 at 02:03

Please note that terms and conditions apply.
Modelling the optical properties of lossless multilayered spheres

Hao Zhang¹, Pengfei Zhu², Yuchen Xu¹, Heyuan Zhu¹ and Min Xu¹

¹ Key Laboratory for Microphotonic and Nanophotonic Structures (Ministry of Education), Department of Optical Science and Engineering, Fudan University, Shanghai 200433, People’s Republic of China
² College of Fundamental Studies, Shanghai University of Engineering Science, Shanghai 201620, People’s Republic of China

E-mail: minx@fudan.edu.cn

Received 5 July 2011, accepted for publication 28 July 2011
Published 18 August 2011
Online at stacks.iop.org/JOpt/13/095704

Abstract
Improved formulae for the Mie scattering of three-layered spheres are presented. By means of comparisons between the scattering cross-sections of three-layered spheres and two-layered spheres, the influence of Mie resonances on the curves of the scattering cross-sections when varying the radius of the core layer is investigated. The transport properties of random media composed of two-layered spheres are investigated as well.

Keywords: scattering, diffusion, propagation

(Some figures in this article are in colour only in the electronic version)

1. Introduction

In recent years, there has been growing interest in studying the propagations of electromagnetic waves in random media composed of small particles [1]. Some experiment results have revealed the optical features associated with the random systems, e.g. the enhanced coherent backscattering in weak localization regions [2, 3], and the non-classical diffusion properties in the critical regime [4]. However, it is very difficult to realize Anderson localization of light in experiments [4], due to the absorption of dielectric particles. Recently, a strongly disordered medium composed of multilayered dielectric spheres constructed from high-index and lossless materials was proposed as a potential random system for realizing Anderson localization of light in [5]. In such a random medium, the diffusion of light is suppressed due to the strongly scattering properties of constituent spheres, according to the fact that the diffusion coefficient \( D = \frac{v_e l^*}{3} \) decreases where resonant scattering effects occur, which decrease the velocity of the electromagnetic energy \( v_e \) and transport mean free path \( l^* \) [18]. It is obvious that scattering of light by multilayered spheres plays an important role in describing the transport properties of light within strongly disordered media.

Numerical calculations of light scattering by multilayered spheres are required in many scientific and applied problems, for example, the optical properties of interstellar grains which are probably composed of layered particles caused by the formation of dust grains in circumstellar environments [6, 7]. Recently there has been considerable improvement in the analysis of light scattering by multilayered spheres using a recursive algorithm based on the Mie scattering theory [8–10].

The Mie theory was originally introduced by Mie in 1908 and Debye in 1909, and describes the interaction of a plane monochromatic wave with a homogeneous sphere having an arbitrary diameter. The detailed description of the Mie scattering can be found elsewhere [11–13].

Much progress has been made in improving the algorithm for Mie scattering since then. In order to handle the numerical instability problems of Riccati–Bessel functions that appear in the formulae for scattering amplitudes and derived physical quantities, e.g., scattering cross-sections [13], Dave introduced a logarithmic derivative of the Riccati–Bessel functions, which behaves well even for highly absorptive spheres with large size [14]. Another major contribution to the stable algorithm of Mie scattering was made by Wiscombe and Wu, who discussed in detail the asymptotic forms of logarithmic derivatives and ratios of Riccati–Bessel functions [15, 8].

In this paper, we present an improved Mie algorithm, and investigate in detail the optical properties of two- and three-layered spheres. Finally we theoretically investigate the...
transport properties of light propagating within random media composed of two- and three-layered spheres.

2. Theory

According to Mie theory, the electromagnetic waves scattered by multilayered spheres can be expressed in terms of vectorial spherical harmonic functions [13]. For simplicity, systems with three- or two-layered spheres are considered in this paper, and more general cases can be investigated in a similar way. Following the standard derivation of the Mie theory described in [13], we obtained the scattering coefficients for the outward electromagnetic waves scattered by the three-layered sphere as

\[
a_n = \frac{(\tilde{T}_n/m_3 + n/z)\psi_n(z) - \psi_{n-1}(z)}{(\tilde{T}_n/m_3 + n/z)\xi_n(z) - \xi_{n-1}(z)}
\]

(1)

\[
b_n = \frac{(\tilde{R}_n/m_3 + n/z)\psi_n(z) - \psi_{n-1}(z)}{(\tilde{R}_n/m_3 + n/z)\xi_n(z) - \xi_{n-1}(z)}
\]

(2)

where \(m_3\) is \(n_3/n_0\) and \(z = kr_3\). \(n_i\) is the refractive index of the \(i\)th layer of the sphere and \(n_0\) is the refractive index of the surrounding medium. \(k (=2\pi/\lambda)\) is the wavenumber of light in the surrounding medium, and \(r_3\) is the radius of the third layer. \(\tilde{R}_n\) and \(\tilde{T}_n\) are defined as

\[
\tilde{R}_n = \frac{\psi'_n(m_3^2z) - A_n^{(3)}\chi_n(m_3^2z)}{\psi_n(m_3^2z) - A_n^{(3)}\chi_n(m_3^2z)}
\]

(3)

\[
\tilde{T}_n = \frac{\psi'_n(m_3^2z) - B_n^{(3)}\chi_n(m_3^2z)}{\psi_n(m_3^2z) - B_n^{(3)}\chi_n(m_3^2z)}
\]

(4)

where \(\psi_n(z)\), \(\chi_n(z)\) and \(\xi_n(z)\) are kinds of Riccati–Bessel functions; \(A_n^{(3)}\) and \(B_n^{(3)}\) are the corresponding coefficients of the expansion functions of the electromagnetic field within the third layer, which can be analytically obtained by a standard derivation according to the Mie theory.

Although the equations for the scattering coefficients seem to be straightforward, one should exercise great caution in the computations of the Riccati–Bessel functions \(\psi_n(z)\), \(\chi_n(z)\), and \(\xi_n(z)\), and ratios of Riccati–Bessel functions, especially in the cases of handling strongly absorbing spheres. In order to obtain the scattering coefficients, we rewrite equations (3) and (4) by introducing \(L_n^{(2)}\) which is defined as

\[
L_n^{(2)}(\rho) = \frac{\chi_n(\rho)}{\psi_n(\rho)} = \frac{\chi_n(\rho)/\psi_n(\rho)}{\chi_n(\rho)/\psi_n(\rho)} = \frac{G_n^{(2)}(\rho)}{H_n(\rho)}
\]

(5)

where the ratios of \(G_n^{(2)}(\rho) = \chi_n(\rho)/\psi_n(\rho)\) and \(H_n(\rho) = \chi_n(\rho)/\psi_n(\rho)\) have been discussed in a series of theoretical works [14, 15, 8–10]. By using the iterative algorithm of \(G_n^{(2)}(\rho)\) in [14, 15], the numerical results for \(L_n^{(2)}(\rho)\) are shown in tables 1 and 2, compared with those for \(L_n^{(1)}(\rho) = \frac{\chi_n(\rho)}{\psi_n(\rho)}\). It is obvious from the numerical comparison, for the lossless spheres, that both \(L_n^{(1)}\) and \(L_n^{(2)}\) are stable. However, when \(n \geq N_{\text{dop}}\), \(L_n^{(1)}(\rho)\) shows numerical instabilities for \(\rho\) with a considerable imaginary part, which means that \(L_n^{(1)}(\rho)\) is a more stable algorithm for calculating the scattering coefficients for spheres composed of dispersive material.

3. Discussion

3.1. Optical properties of multilayered spheres

In this section, the influences on the scattering cross-section of the radii of the inner shells of three- and two-layered spheres are investigated. In figure 1, the fixed parameters of the three-layered spheres are \(n_1 = 2.7\) (TiO₂), \(n_2 = 1.44\) (silica), \(n_3 = 2.7\) and \(r_3 = 600\) nm.

It is shown in figures 1(a)–(e) that the calculated scattering cross-sections manifest themselves as spectra consisting of rapid oscillations superimposed on slowly varying profiles, which are the standard configurations of the Mie scatterings [13]. The positions of the resonant peaks, which are determined by the dominant components of the scattering coefficients, i.e. \(a_n\) and \(b_n\), are sensitive to the morphology of the spheres [16]. Since each of the scattering coefficients \(a_n\) and \(b_n\) is associated with a special vectorial spherical function, the dominant component would indicate the corresponding dominant electromagnetic field mode distributed within the sphere.

In figures 1(a)–(c), the radius of the second shell is fixed to be \(r_2 = 400\) nm, and the radius of the first shell is changed from \(r_1 = 50\) to \(r_1 = 300\) nm. It is shown that the rough profiles of the curves and positions of the Mie resonance remain fixed when \(r_1\) changes from \(r_1 = r_2/6\) to \(r_1/3\), which means that the dominant field modes within the spheres do not change significantly in these situations. It is due to the fact that it is less possible for electromagnetic waves with wavelength several times longer than \(r_1\) to form resonant modes inside the core layer. In figure 1(c) where \(r_1 = 300\) nm, while the slowly varying profile remains similar, the positions of the Mie resonance change obviously, especially in the region with

<table>
<thead>
<tr>
<th>(n)</th>
<th>(L_n^{(1)}(\rho) = \frac{\chi_n(\rho)}{\psi_n(\rho)})</th>
<th>(L_n^{(2)}(\rho) = \frac{G_n^{(2)}(\rho)}{H_n(\rho)})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>((-1.0000, 0.0000))</td>
<td>((-1.0000, 0.0000))</td>
</tr>
<tr>
<td>100</td>
<td>((-0.9999, 0.0001))</td>
<td>((-0.9999, 0.0001))</td>
</tr>
<tr>
<td>200</td>
<td>((-0.9997, 0.0003))</td>
<td>((-0.9997, 0.0003))</td>
</tr>
<tr>
<td>300</td>
<td>((-0.9995, 0.0005))</td>
<td>((-0.9995, 0.0005))</td>
</tr>
<tr>
<td>400</td>
<td>((-0.9993, 0.0007))</td>
<td>((-0.9993, 0.0007))</td>
</tr>
<tr>
<td>500</td>
<td>((-0.9991, 0.0009))</td>
<td>((-0.9991, 0.0009))</td>
</tr>
<tr>
<td>535</td>
<td>((-0.9988, 0.0011))</td>
<td>((-0.9988, 0.0011))</td>
</tr>
<tr>
<td>550</td>
<td>((-0.9986, 0.0013))</td>
<td>((-0.9986, 0.0013))</td>
</tr>
</tbody>
</table>

Table 2. Asymptotic behaviours of \(L_n(\rho)\) (note: \(n = 500, 15\), \(N_{\text{dop}} = 569\)).

<table>
<thead>
<tr>
<th>(n)</th>
<th>(L_n^{(1)}(\rho) = \frac{\chi_n(\rho)}{\psi_n(\rho)})</th>
<th>(L_n^{(2)}(\rho) = \frac{G_n^{(2)}(\rho)}{H_n(\rho)})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>((-1.0000, 0.0000))</td>
<td>((-1.0000, 0.0000))</td>
</tr>
<tr>
<td>100</td>
<td>((-0.9999, 0.0001))</td>
<td>((-0.9999, 0.0001))</td>
</tr>
<tr>
<td>200</td>
<td>((-0.9997, 0.0003))</td>
<td>((-0.9997, 0.0003))</td>
</tr>
<tr>
<td>300</td>
<td>((-0.9995, 0.0005))</td>
<td>((-0.9995, 0.0005))</td>
</tr>
<tr>
<td>400</td>
<td>((-0.9993, 0.0007))</td>
<td>((-0.9993, 0.0007))</td>
</tr>
<tr>
<td>500</td>
<td>((-0.9991, 0.0009))</td>
<td>((-0.9991, 0.0009))</td>
</tr>
<tr>
<td>550</td>
<td>((-0.9988, 0.0011))</td>
<td>((-0.9988, 0.0011))</td>
</tr>
<tr>
<td>569</td>
<td>((-0.9986, 0.0013))</td>
<td>((-0.9986, 0.0013))</td>
</tr>
<tr>
<td>580</td>
<td>((-0.9984, 0.0014))</td>
<td>((-0.9984, 0.0014))</td>
</tr>
</tbody>
</table>

Table 1. Asymptotic behaviours of \(L_n(\rho)\) (note: \(n = 500, 0\), \(N_{\text{dop}} = 535\)).
Figure 1. Scattering cross-section $\sigma$ for three-layered spheres with $r_3 = 600$ nm and $n_1 = 2.7$, $n_2 = 1.44$, $n_3 = 2.7$, except that (a) $r_1 = 50$ nm, $r_2 = 400$ nm, (b) $r_1 = 200$ nm, $r_2 = 400$ nm, (c) $r_1 = 300$ nm, $r_2 = 400$ nm, (d) $r_1 = 200$ nm, $r_2 = 300$ nm, and (e) $r_1 = 200$ nm, $r_2 = 500$ nm.

wavelength close to $r_1$, where more complex structure appears. This suggests that new resonant modes may form inside the spheres with a larger $r_1$.

In figures 1(e), (b) and (d), the radius of the first shell is fixed to be $r_1 = 200$ nm, and the radius of the second shell is changed from $r_2 = 300$ to 500 nm. It is shown that the rough profiles of the curves and positions of the Mie resonances make a significant difference for changing $r_2$. By comparison with the results obtained from changing $r_1$, it is obvious that the scattering at the second shell influences more greatly the total scattering cross-section, and the resonant modes are easier to change by varying $r_2$. This feature of the three-layered sphere can be used in designing systems with desired properties.

In figure 2, the outer radius of the two-layered sphere is fixed to be $r_2 = 600$ nm, and the refractive indices are $n_1 = 1.44$ and $n_2 = 2.7$, while the radius of the first shell $r_1$ varies from 100 to 500 nm with an interval of 100 nm.

From figures 2(a) to (c) where $r_1 \leq 300$ nm, the positions of Mie resonance coincide correspondingly, though the profiles change obviously, which means that the resonant modes inside the spheres are similar in these situations. While in the cases of figures 2(c)–(e) where $r_1 \geq 300$ nm, both the slowly varying profiles and the positions of the peaks make significant differences, which suggests that different resonant modes form inside the spheres.

In addition, it should be noted that the difference between the sphere investigated in figure 1(e) and that in 2(e) is the additional core layer with $r_1 = 200$ nm, $n_1 = 2.7$ in the former case. The great similarity of these two curves suggests that a wave with wavelength much longer than the size of the core layer is not sensitive to the existence of a core layer. Such a similarity can be found between figures 1(a) and 2(d) as well.

However, if a wave with wavelength $\lambda$ compared to the size of the spheres propagates within a random medium consisting of two-layered spheres, Mie scatterings take place and the calculated effective dielectric constants show different characteristics.

3.2. The transport velocity of the electromagnetic energy

According to the numerical results shown in figures 1 and 2, the scattering from a core layer with a radius quite a bit smaller than the incident wavelength, i.e. $r_1 \ll \lambda$, contributes trivially to the total scattering cross-section of multilayered spheres. The influence of the structure of multilayered spheres on transport properties of light propagating within random media composed of multilayered spheres is shown in figure 3. For simplicity, we only consider two-layered-sphere random media herein.

Since the random medium with a volume fraction $f = 30\%$ investigated in figure 3 is not dense, the localization theory in [17] can be used to calculate the transport velocities of the electromagnetic energy of light propagating within such random media. The blue (top solid line) and red lines indicate the calculated transport velocities for random media composed of pure silica and TiO$_2$ spheres, respectively. The three black lines correspond to random media composed of two-layered spheres with a silica core and a TiO$_2$ mantle. Comparing figure 3 with 2, it is obvious that the valleys of $v_e$ are in coincidence with the peaks of $\sigma$, which is due to the fact that the slowdown of $v_e$ is caused by the dwell
Figure 2. Scattering cross-section $\sigma_s$ for two-layered spheres. The radius of the second shell is fixed to be $r_2 = 600$ nm, and the refractive indices of the core layer and the second layer are $n_1 = 1.44$, $n_2 = 2.7$ respectively. The values of $r_1$ are (a) $r_1 = 100$ nm, (b) $r_1 = 200$ nm, (c) $r_1 = 300$ nm, (d) $r_1 = 400$ nm, (e) $r_1 = 500$ nm.

Figure 3. Velocity of the electromagnetic energy $v_e$ for random media composed of pure and two-layered spheres. The volume fraction of the spheres is $f = 30\%$.

4. Summary

In conclusion, we have provided an improvement for the calculations of the Mie scattering of multilayered spheres. The scattering cross-sections for three- and two-layered spheres are investigated in detail and we find that a core layer with size smaller than the wavelength of the incident light has little influence on the total scattering cross-section, which suggests that resonant modes cannot form inside such a core layer. A numerical comparison for random media composed of two-layered spheres and pure spheres is performed, and we find that the disorder of random systems increases where Mie resonances of spheres occur, which can lead to a good choice for designing random lasers working with localized modes.

Acknowledgments

This work was supported by the Absorption and Innovation Projects of Introduced Technology of Shanghai (No. 2010CH-007), and the exchange-scholar programme of the University of...
Konstanz and Fudan University. Furthermore, we acknowledge fruitful discussions with Georg Maret, Christof Aegerter and Wolfgang Bührer.

References