Developing a Composite Trust Model for Multi-agent Systems

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ABSTRACT
The Semantic Web, conceived as a collection of agents, brings new opportunities and challenges to trust research. Enabling trust to ensure more effective and efficient interaction is at the heart of the Semantic Web vision. We propose a composite trust model based on statistical decision theory and Bayesian sequential analysis to balance the costs and benefits during the process of trust evaluating and combine a variety of sources of information to assist users with making the correct decisions in selecting the appropriate service providers according to their preferences. The model proposed by this paper gives trust a strict mathematical interpretation in terms of probability theory.

Categories and Subject Descriptors
I.2.11 [**Artificial Intelligence**]: Distributed Artificial Intelligence – *Intelligent agents, Multiagent systems*; F.1.2 [**Computation by Abstract Devices**]: Modes of Computation – *Probabilistic computation.*

General Terms
Theory, Security, Algorithms.

Keywords
Trust Model, Semantic Web, Bayesian Sequential Analysis.

1. INTRODUCTION
In our model, the quality of a provider agent can be considered to be an unknown numerical quantity, and will represent it by θ and it is possible to treat θ as a random quantity with a probability distribution. Consider the situation of an agent *A* try to make an estimate of trust value for provider agent *B*. *A* holds a prior information (subjective) of *B*, represented by distribution $\pi(\theta)$, and request *A*'s acquaintance to give opinions on *B*'s quality. After *A* receives the assessments of *B*'s quality from his acquaintances, *A* takes these statements as sample about θ. Outcome of these sample is a random variable and will be denoted $\boldsymbol{X}$. A particular realization of $\boldsymbol{X}$ will be denoted x and $\boldsymbol{X}$ will be assumed to be a random variable, with density $f(x|\theta)$. Then, we can compute "posterior distribution" of θ given x, denoted $\pi(\theta|x)$. $\pi(\theta|x)$ reflects the update beliefs about θ after (posterior to) observing the sample x. We take the posterior distribution of θ, $\pi(\theta|x)$, as the estimate of *B*'s trust. If we want to take further investigation on *B*'s quality for more accuracy, $\pi(\theta|x)$ will be used as prior distribution for the next stage of investigation instead of original $\pi(\theta)$.

When several provider agents exist, *A* need to decide which one should be selected. At that time, the preferences of agent *A*'s owner should be considered properly to make this decision. Therefore, *utility function* should be constructed for agent *A*'s owner, which represented by $U_A(r)$, to express his preferences, where r represents rewards of the consequences of a decision. Supposing that $\pi(\theta|x)$ is the posterior distribution of agent *B*, the expected utility of function $U_A(r)$ over $\pi(\theta|x)$, denoted $E^{\pi(\theta|x)}[U_A(r)]$, is possible gain of consequence of selecting *B*.

2. COMPOSITE TRUST MODEL
By treating an agent as a node, the "knows" relationship as an edge, a directed graph emerges. To facilitate the model description, agents and their environment are to be defined. To clarify the idea of our trust model, we begin with a simple illustration. Consider the scenario that agent *A* is evaluating trust value of *B* and *C* for being business. The set of all consultant agents that *A* requests for this evaluation as well as *A*, *B*, *C* can be considered to be a unique society of agents. In our example (see Figure 1) it is called a "closed society of agents" with respect to *A*.

2.1 Closed Trust Model
We will treat the service quality, θ, as a continuous variable here, and suppose that the service charges of *B* and *C* are 250 and 210 units respectively (SC_B = 250, SC_C = 210, where *SC* denotes *service charge*). As shown in Figure 1, the agent *A* feels that θ_B, the service quality of *B*, has a normal prior density, $N(0.64, (0.08)^2)$. We also suppose that the prior density of θ_C is $N(0.5, (0.15)^2)$ here. The probability distribution of $\boldsymbol{X}$ that represents the assessments of service quality from consultant agents will, of course, depend upon the unknown state of nature θ. Therefore, we assume that $\boldsymbol{X}$ is another continuous random variable with density $f(x|\theta) \sim N(\theta, (0.05)^2)$. We also assume that the agent *A*'s owner has a utility function $U_A(r) = 208.4 + 459.8r - 17.95r^2 - 99.84r^3$.

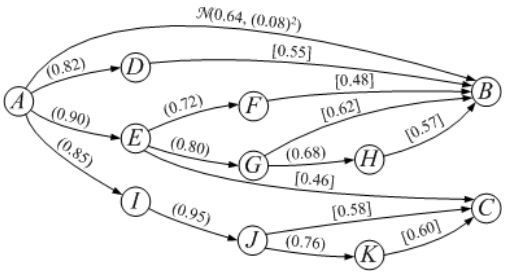

Figure 1. A "closed society of agents" with respect to agent *A*

Assume $X \sim N(\theta, \sigma^2)$, where θ is unknown but σ^2 is known. Let be prior information, $\pi(\theta)$, a $N(\mu, \tau^2)$ density, where μ and τ^2 are known. Note that the marginal distribution of $\boldsymbol{X}$, $m(x)$, is $N(\mu, \sigma^2 + \tau^2)$ and the posterior distribution θ of give x is $N(\mu(x), \rho)$, where

$$\mu(x) = \frac{\sigma^2}{\sigma^2 + \tau^2}\mu + \frac{\tau^2}{\sigma^2 + \tau^2}x \qquad \rho = \frac{\tau^2\sigma^2}{\tau^2 + \sigma^2} \qquad \textbf{(1)}$$

We take the assessments of *B*'s quality from the consultant agents as sample about θ_B and combine these sample information (x) and prior information into posterior distribution of θ_B given x. In order to answer the question of which sample information should be used with higher priority, we propose following order rules: (1) The sample from the agent with shorter referral distance should be used first; (2) If the samples come from the agents that have the same referral distance, which with larger reliability factor is prior to that with smaller one.

Now, we can evaluate trust value of *B* for *A* by using above formulas and information. Firstly, we use the sample information from *D* through the path *A*→*D*→*B*, and the posterior distribution $\pi_B(\theta|x = 0.55) \sim N(0.5753, (0.0424)^2)$, where

$$\frac{0.05^2}{0.08^2 + 0.05^2} \times 0.64 + \frac{0.08^2}{0.08^2 + 0.05^2} \times 0.55 = 0.5753$$

$$\frac{0.08^2 \times 0.05^2}{0.08^2 + 0.05^2} = 0.0424^2$$

However, the reliability factor has not been considered during the process of calculating this posterior distribution. We used following formula to rectify above $\pi_B(\theta|x = 0.55)$, where p_{old}, p and p_{new} represent the parameter of prior distribution, the posterior distribution before rectification and the posterior distribution after rectification respectively (p is mean or variant for normal distribution), and R is reliability factor.

$$p_{new} = p_{old} + (p - p_{old}) \times R \qquad \textbf{(2)}$$

Hence, after rectification, $\mu_B = 0.64 + (0.5753 - 0.64) \times 0.82 = 0.5869$ and $\tau_B = 0.08 + (0.0424 - 0.08) \times 0.82 = 0.0492$. Then the posterior distribution $\pi_B(\theta|x = 0.55)$ is $N(0.5869, (0.0492)^2)$. The residual calculating process of *B* and the whole the process of *C* are shown in Table 1 and 2. Note that we employ multiplying to merge two or more than two reliability factors. The reason behind using multiplying is that if the statement is true only if the agents who propagate this statement all tell the truth and it is considered to be independent for any two agents to lie or not to lie. Expected utility of *B* is:

$$\begin{aligned}\text{utility of } B &= \int_{-\infty}^{+\infty} U_A(r)\pi_B(\theta \mid x)d\theta - SC_B \\ &= \int_{-\infty}^{+\infty}(208.4 + 459.8\theta - 17.95\theta^2 - 99.84\theta^3) \times \\ &\quad \frac{1}{\sqrt{2\pi} \times 0.0309} e^{-\frac{(\theta - 0.5696)^2}{2 \times 0.0309^2}} d\theta - 250 \\ &= 195.82\end{aligned}$$

$$\begin{aligned}\text{and utility of } C &= \int_{-\infty}^{+\infty} U_A(r)\pi_C(\theta \mid x)d\theta - SC_C \\ &= 225.34\end{aligned}$$

hence (225.34 > 195.82), *C* is more appropriate than *B* in the eyes of *A* .

2.2 Open Trust Model

Above discussion is under the condition that the "closed society of agents" must be defined at first, but it is nearly impossible for inherent open and dynamic Web. Our idea is that at every stage of the procedure (after every given observation) one should compare the (posterior) utility of making an immediate decision with the "expected" (preprosterior) utility that will be obtained if more observations are taken. If it is cheaper to stop and make a decision, that is what should be done.

The goal of preposterior analysis is to choose the way of investigation which minimizes overall cost. This overall cost consists of the decision loss (*opportunity cost*) and the cost of conducting observation (*consultant fee*). Note that these two quantities are in opposition to each other, we propose an approach to balance these two costs.

Table 1. The process of evaluating *B*'s trust

Step	Path	Statement	Reliability Factor	Prior Distribution		Posterior Distribution		Utility
				μ	τ	μ	τ	
1	A→D→B	0.55	0.8200	0.6400	0.0800	0.5869	0.0492	201.39
2-1	A→E→G→B	0.62	0.7200	0.5869	0.0492	0.5986	0.0390	205.46
2-2	A→E→F→B	0.48	0.6480	0.5986	0.0390	0.5695	0.0337	195.75
3	A→E→G→H→B	0.57	0.4896	0.5695	0.0337	0.5696	0.0309	195.82

Table 2. The process of evaluating *C*'s trust

Step	Path	Statement	Reliability Factor	Prior Distribution		Posterior Distribution		Utility
				μ	τ	μ	τ	
1	A→E→C	0.46	0.9000	0.5000	0.1500	0.4676	0.0577	198.72
2	A→I→J→C	0.58	0.8075	0.4676	0.0577	0.5194	0.0416	218.06
3	A→I→J→K→C	0.60	0.6137	0.5194	0.0416	0.5396	0.0357	225.34

Table 3. The process of Bayesian sequential analysis

Stage	Agent *B*				Agent *C*				Consultant Fee	Utility		Decision
	Prior Distribution		Marginal Distribution		Prior Distribution		Marginal Distribution			Immediate Decision	Further Investigation	
	μ_B	τ_B	$\mu_{x\|B}$	$\tau_{x\|B}$	μ_C	τ_C	$\mu_{x\|C}$	$\tau_{x\|C}$				
1	0.6400	0.0800	0.6400	0.0943	0.5000	0.1500	–	–	3	217.78	219.24	Continue
2	0.5869	0.0492	0.5869	0.0701	0.4676	0.0577	–	–	3	201.39	201.80	Continue
3	0.5695	0.0337	–	–	0.5194	0.0416	0.5194	0.0650	2	218.06	216.09	Stop

We continue above example used in the illustration of the closed trust model. As shown in Figure 2, we begin at the stage 1 when *A* only holds the prior information of *B* and has no any information about *C* (even the existence of *C*, but it is more likely that an agent with the prior distribution of $N(0.5, (0.15)^2)$ and the expected service charge of 210 is near in the network). Agent *A* either can make an immediate decision (to select *B*) or can send request to his acquaintances for their opinions by extending the tree of Figure 2 down to the next layer.

Suppose that the cost of consultant services is determined by how much agents will be requested at the next stage and the consultant fee is the constant of 1 for each times (for example, at the stage 1, agent *A* can ask Agent *D*, *E* and *I* for their opinions, so the consultant fee of the stage 1 will be 3).

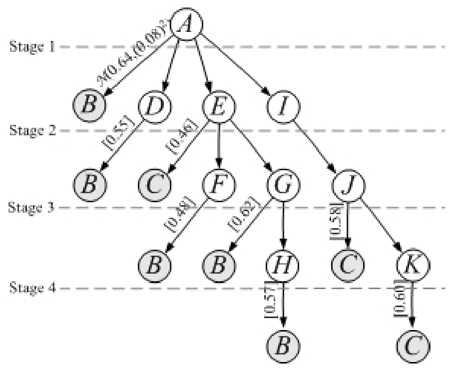

Figure 2. The process of trust evaluating

The utility of an immediate decision is the larger of

$$\int_{-\infty}^{+\infty} U_A(r)\pi_B(\theta)d\theta - SC_B = 217.78$$

here, $\pi_B(\theta) \sim N(0.64, (0.08)^2)$.

and $\int_{-\infty}^{+\infty} U_A(r)\pi_C(\theta)d\theta - SC_C = 207.54$

here, $\pi_C(\theta) \sim N(0.5, (0.15)^2)$.

Hence the utility of an immediate decision is 217.78.

If the request message is sent and *x* observed, the posterior density $\pi_B(\theta|x)$, is $N(\mu(x), \rho)$, where

$$\mu(x) = \frac{0.05^2}{0.08^2 + 0.05^2} \times (0.64) + \frac{0.08^2}{0.08^2 + 0.05^2}(x) \cong 0.1798 + (0.7191)x$$

$$\rho = \frac{0.08^2 \times 0.05^2}{0.08^2 + 0.05^2} \cong 0.0018$$

However, that we do not know which *x* will occur, but we know the relevant distribution for ***X*** is the "predictive" or marginal distribution, $m_B(x)$, which in this situation is $N(0.64, (0.08)^2 + (0.05)^2)$. Note that if $x < 0.5914$ is observed, the expected utility of $\pi_B(\theta|x)$ is less than 207.54, so we prefer to select *C* instead of *B*. Hence expected utility of not making immediate decision is

$$\int_{-\infty}^{0.5914} 207.54\, m_C(x)dx +$$

$$\int_{0.5914}^{+\infty} \left(\int_{-\infty}^{+\infty} U_A(r)\pi_B(\theta \mid x)d\theta - SC_B\right)m_B(x)dx - 3$$

$$= 219.24$$

This just is the opportunity cost. Because 219.24 > 217.78, and then further investigation would be well worth the money, in other words, *A* should send request to his acquaintances for their opinions. The residual process of Bayesian sequential analysis are shown in Table 3 and remember that the further exploiting should be terminated immediately along the path on which a cycle is detected. As shown in Table 3, at the stage 3, the expected utility of *C* begins to larger than that of *B*, and because 218.06 > 216.09, making an immediate decision is more profitable. Therefore, *A* should stop investigating and select *C* as a decision. The advantage of sequential analysis should be clear now. It allows one to gather exactly the correct amount of data needed for a decision of the desired accuracy.

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