Technical Appendix
for “Rotten Parents and Disciplined Children: A Politico-Economic Theory of Public Expenditure and Debt”

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C Technical Appendix

C.1 Analysis of Example II, section 3.2

Proposition 6 *Assume that* $\theta < \frac{1+\beta}{(1+\lambda)\beta}$ *and* $R \in \left[1 + \frac{1+\psi}{(1+\lambda)\beta}, \frac{1+\beta+\theta(1+\psi)}{((1+\lambda)\theta+(1+\beta)\lambda)\beta}\right]$*. Then, there exists* $\underline{b} < b_0^*$ *such that for* $b \in [\underline{b}, \bar{b}]$*, a Markov equilibrium is characterized by the following policy functions:*

$$\tau = T(b) \equiv \begin{cases} \bar{\tau} - \frac{R(1+\beta)}{w(1+\beta+\theta(1+\psi))}(b_0^* - b) & \text{if } b \in [\underline{b}, b_0^*) \\ \bar{\tau} & \text{otherwise} \end{cases}, \tag{64}$$

$$g = G(b) \equiv \begin{cases} g_0^* + \frac{\theta(1+\psi)R}{1+\beta+\theta(1+\psi)}(b_0^* - b) & \text{if } b \in [\underline{b}, b_0^*) \\ b_n^* + \bar{\tau}w - Rb & \text{if } b \in \left[b_n^*, b_{n+1}^*\right) \end{cases}, \tag{65}$$

$$b' = B(b) \equiv \begin{cases} b_0^* \equiv \bar{b}\left(1 - \frac{\theta(1+\psi)(1-\bar{\tau})}{\bar{\tau}(1+\beta)}\right) & \text{if } b \in [\underline{b}, b_1^*) \\ b_n^* & \text{if } b \in \left[b_n^*, b_{n+1}^*\right] \end{cases}, \tag{66}$$

where $\bar{b} \equiv \bar{\tau}w/(R-1)$, $g_0^* \equiv w\theta(1+\psi)(1-\bar{\tau})/(1+\beta) > 0$, *and the sequence* $\{b_n^*\}_{n=0,1,2,..,\infty}$ *is the unique solution to the difference equation*

$$\left(b_n^* - b_{n+1}^* + \bar{\tau}w\right)^{1+\psi}\left(b_n^* - Rb_n^* + \bar{\tau}w\right)^{(1+\lambda)\beta} = \left(b_{n+1}^* - Rb_{n+1}^* + \bar{\tau}w\right)^{1+\psi+(1+\lambda)\beta}, \tag{67}$$

given b_0^**. The sequence* $\{b_n^*\}_{n=0,1,2,..,\infty}$ *is monotonically increasing in* n *and* $\lim_{n\to\infty} b_n^* = \bar{b}$.

The equilibrium is shown in Figure A1.

FIGURE A1 HERE

Proof Strategy. The proof strategy is analogous to Krusell and Smith (2003). We have structured the proof in 10 lemmas. Lemmas 6 establishes that the set of admissible interest rates, $R \in \left[1 + \frac{1+\psi}{(1+\lambda)\beta}, \frac{1+\beta+\theta(1+\psi)}{((1+\lambda)\theta+(1+\beta)\lambda)\beta}\right]$ is nonempty. Lemma 7 establishes that the sequence $\{b_n^*\}_{n=0}^{\infty}$ implicitly defined by (67) describing the set of steady states converges to $\bar{b}$ along an increasing path. Lemma 8-15 jointly establish that there exist no profitable one-period deviation from the equilibrium. In other words, they show that if future policy outcomes follow the equilibrium functions (64), (65) and (66), the current political choice is also characterized by (64), (65) and (66) (namely, we have a fixed point). Specifically, Lemma 8 checks the intra-temporal optimality condition and Lemma 9 provides the objective functions of the current government's intertemporal choices. Lemma 10 proves then that there exists $\underline{b} < b_0^*$ such that for $b \in [\underline{b}, b_0^*]$, b_0^* and b_n^* are locally optimal for $b' \in [\underline{b}, b_0^*)$ and $b' \in \left[b_n^*, b_{n+1}^*\right)$, respectively. Next, Lemma 13 shows that b_0^* is globally optimal for $b \in [\underline{b}, b_0^*]$. We then move to $b \in [b_0^*, b_1^*]$. The local optimality of b_n^* for $b' \in \left[b_n^*, b_{n+1}^*\right)$ is obtained by Lemma 11. The global optimality of $b' = b_0^*$ is ensured by Lemma 12 and 15, showing that $b' = b_0^*$ is better than any $b' > b_0^*$ and any $b' < b_0^*$, respectively. Using Lemma 14, the same logic applies for $b \in \left[b_n^*, b_{n+1}^*\right]$ with $n \geq 1$.

Lemma 6 $\left[1 + \frac{1+\psi}{(1+\lambda)\beta}, \frac{1+\beta+\theta(1+\psi)}{((1+\lambda)\theta+(1+\beta)\lambda)\beta}\right]$ *is nonempty if* $\theta < \frac{1+\beta}{(1+\lambda)\beta}$.

Proof. Some algebra establishes that if $\theta < \frac{1+\beta}{(1+\lambda)\beta}$,

$$\frac{1+\beta+\theta(1+\psi)}{((1+\lambda)\theta+(1+\beta)\lambda)\beta} > 1 + \frac{1+\psi}{(1+\lambda)\beta}$$

always holds. ■

Lemma 7 *The sequence* $\{b_n^*\}_{n=0}^{\infty}$ *converges to* $\bar{b}$ *along an increasing path.*

Proof. (67) gives an implicit difference equation of b_n^*. Rearranging (67) and using the fact that $\bar{\tau} w = (R-1)\bar{b}$, we obtain

$$y_n = (x_n)^{-\frac{(1+\lambda)\beta}{1+\psi}}, \tag{68}$$

where $y_n \equiv \frac{(R-1)\bar{b}+b_n^*-Rb_{n+1}^*}{(R-1)\left(\bar{b}-b_{n+1}^*\right)}$ and $x_n \equiv \frac{\bar{b}-b_n^*}{\bar{b}-b_{n+1}^*}$. Linearizing (68) around $b_n^* = b_{n+1}^*$ yields

$$y_n - 1 = -\frac{(1+\lambda)\,\beta}{1+\psi}\,(x_n - 1)\,,$$

or equivalently

$$\frac{b_n^* - b_{n+1}^*}{(R-1)\left(\bar{b} - b_{n+1}^*\right)} = \frac{(1+\lambda)\,\beta}{1+\psi}\frac{\left(b_n^* - b_{n+1}^*\right)}{\bar{b} - b_n^*}.$$

This establishes

$$b_{n+1}^* = \bar{b} - \frac{1+\psi}{(1+\lambda)\,\beta\,(R-1)}\left(\bar{b} - b_n^*\right). \tag{69}$$

It is immediate that if $\frac{1+\psi}{(1+\lambda)\beta(R-1)} < 1$ (or equivalently $R > 1 + \frac{1+\psi}{(1+\lambda)\beta}$), b_n^* is converging to the maximum debt level $\bar{b}$ along an increasing path. ■

Lemma 8 *Suppose that* $B(b)$ *follows (66). Then, the optimal intra-temporal solution is such that* $\tau = 1 - \frac{1+\beta}{\theta(1+\psi)w} g \leq \bar{\tau}$ *if* $b \leq b_0^*$ *and* $\tau = \bar{\tau}$ *otherwise.*

Proof. The first-order condition linking τ and g establishes

$$\tau = \begin{cases} 1 - \frac{1+\beta}{\theta(1+\psi)w} g & \text{if } g \geq \frac{(1-\bar{\tau})\theta(1+\psi)w}{1+\beta} \\ \bar{\tau} & \text{otherwise} \end{cases}.$$

Given $B(b)$, the above equality leads to $G(b)$. Replacing g with $G(b)$, we obtain an equivalence between $g \geq \frac{(1-\bar{\tau})\theta(1+\psi)w}{1+\beta}$ and $b \leq b_0^*$. ■

Lemma 9 *Suppose that future policy outcomes follow (64), (65) and (66). Then, the current government's objective function is*

$$V\left(b';b\right) = \begin{cases} \begin{array}{c} (1+\beta+\theta(1+\psi))\log(b'+w-Rb) \\ +((1+\lambda)\theta+(1+\beta)\lambda)\beta\log(b_0^*+w-Rb') + \theta\zeta\beta\lambda\log(\bar{\tau}w-(R-1)b_0^*) \end{array} & \textit{if } b' \in [\underline{b}, b_0^*] \\ \begin{array}{c} (1+\beta+\theta(1+\psi))\log(b'+w-Rb) \\ +(1+\lambda)\theta\beta\log(b_n^*-Rb'+\bar{\tau}w) + \theta\zeta\beta\lambda\log(\bar{\tau}w-(R-1)b_n^*) \end{array} & \textit{if } b' \in \left[b_n^*, b_{n+1}^*\right] \end{cases}, \tag{70}$$

for $b \in [\underline{b}, b_0^*]$ *and*

$$V\left(b';b\right) = \begin{cases} \begin{array}{c} (1+\psi)\,\theta\log(b'+w-Rb) \\ +((1+\lambda)\theta+(1+\beta)\lambda)\beta\log(b_0^*+w-Rb') + \theta\zeta\beta\lambda\log(\bar{\tau}w-(R-1)b_0^*) \end{array} & \textit{if } b' \in [\underline{b}, b_0^*] \\ \begin{array}{c} (1+\psi)\,\theta\log(b'+w-Rb) \\ +(1+\lambda)\theta\beta\log(b_n^*+\bar{\tau}w-Rb') + \theta\zeta\beta\lambda\log(\bar{\tau}w-(R-1)b_n^*) \end{array} & \textit{if } b' \in \left[b_n^*, b_{n+1}^*\right] \end{cases}, \tag{71}$$

for $b \geq b_0^*$, *where* $\zeta \equiv \frac{(1+\lambda)\beta}{1-\beta\lambda}$.

Proof. (64), (65) and (66) establish that for $i \geq 1$, $\tau_{t+i} = \bar{\tau}$ and $b_{t+1+i} = b_n^*$ if $b_{t+1} \in \left[b_n^*, b_{n+1}^*\right]$. If $b_{t+1} \in [\underline{b}, b_0^*]$, we have $\tau_{t+1} = \bar{\tau} - \frac{R(1+\beta)}{w(1+\beta+\theta(1+\psi))}\left(b_0^* - b_{t+1}\right)$, $\tau_{t+i} = \bar{\tau}$ and $b_{t+i} = b_0^*$ for $i \geq 2$. Denoting b' as the current government's choice of public debt and ignoring constant terms, we thus obtain

$$V_O\left(b'\right) = \begin{cases} \left(\left(1+\lambda\right)\theta + \left(1+\beta\right)\lambda\right)\log\left(b_0^* - Rb' + w\right) + \frac{(1+\lambda)\theta}{1-\beta\lambda}\beta\lambda\log\left(\bar{\tau}w - \left(R-1\right)b_0^*\right) & \text{if } b' \in [\underline{b}, b_0^*] \\ \left(1+\lambda\right)\theta\log\left(b_n^* - Rb' + \bar{\tau}w\right) + \frac{(1+\lambda)\theta}{1-\beta\lambda}\beta\lambda\log\left(\bar{\tau}w - \left(R-1\right)b_n^*\right) & \text{if } b' \in \left[b_n^*, b_{n+1}^*\right] \end{cases}.$$

Substituting the above equation into (17) and using Lemma 8 lead to (70) and (71). ■

Lemma 10 *Suppose that future policy outcomes follow (64), (65) and (66). Then, there exists* $\underline{b} < b_0^*$ *such that for* $b \in [\underline{b}, b_0^*]$*, any choice* $b' \in [\underline{b}, b_0^*)$ *can be improved by* $b' = b_0^*$ *and any choice* $b' \in \left(b_n^*, b_{n+1}^*\right)$ *can be improved by* $b' = b_n^*$*.*

Proof. For $b \in [\underline{b}, b_0^*]$, since $\frac{\partial V}{\partial b'}$ is increasing in b and decreasing in b', it is sufficient to show that

$$\left.\frac{\partial V}{\partial b'}\right|_{b=\underline{b}, b'=b_0^*} = \frac{1+\beta+\theta\left(1+\psi\right)}{b_0^* + w - R\underline{b}} - \frac{\left(\left(1+\lambda\right)\theta + \left(1+\beta\right)\lambda\right)\beta R}{b_0^* + w - Rb_0^*} \geq 0.$$

Since $R \leq \frac{1+\beta+\theta(1+\psi)}{((1+\lambda)\theta+(1+\beta)\lambda)\beta}$, there exists $\underline{b} < b_0^*$ such that the above inequality always holds. Hence, any choice $b' \in [\underline{b}, b_0^*]$ can be improved by $b' = b_0^*$.

For $b' \in \left[b_n^*, b_{n+1}^*\right]$, it is sufficient to prove that

$$\left.\frac{\partial V}{\partial b'}\right|_{b=b_0^*, b'=b_n^*} = \frac{1+\beta+\theta\left(1+\psi\right)}{b_n^* + w - Rb_0^*} - \frac{\left(1+\lambda\right)\theta\beta R}{b_n^* + \bar{\tau}w - Rb_n^*} \leq 0.$$

Since $\left.\frac{\partial V}{\partial b'}\right|_{b=b_0^*, b'=b_n^*}$ is decreasing in b_n^*, we only need to show

$$\frac{1+\beta+\theta\left(1+\psi\right)}{b_0^* + w - Rb_0^*} \leq \frac{\theta\left(1+\lambda\right)\beta R}{b_0^* + \bar{\tau}w - Rb_0^*}.$$

The above inequality implies that

$$\begin{aligned} \left(1+\beta+\theta\left(1+\psi\right)\right)\bar{\tau} - \theta\left(1+\lambda\right)\beta R \;&\leq\; \left(1+\beta+\theta\left(1+\psi\right) - \theta\left(1+\lambda\right)\beta R\right)\left(R-1\right)\frac{b_0^*}{w} \\ =\; \left(1+\beta+\theta\left(1+\psi\right) - \theta\left(1+\lambda\right)\beta R\right)&\left(\frac{\left(1+\beta+\theta\left(1+\psi\right)\right)\bar{\tau} - \theta\left(1+\psi\right)}{1+\beta}\right) \end{aligned}$$

First note that when $\bar{\tau} = 1$, LHS is equal to RHS. For the inequality to hold for $\bar{\tau} \in [0, 1)$, we need to show that the slope of LHS $1+\beta+\theta\left(1+\psi\right)$ is greater than the slope of RHS $\frac{(1+\beta+\theta(1+\psi)-\theta(1+\lambda)\beta R)(1+\beta+\theta(1+\psi))}{1+\beta}$. This is ensured by the fact that $R > 1 + \frac{1+\psi}{(1+\lambda)\beta}$. Hence, any $b' \in \left(b_n^*, b_{n+1}^*\right)$ with $n \geq 0$ can be improved by b_n^*. ■

Lemma 11 *Suppose that future policy outcomes follow (64), (65) and (66). Then, for* $b \in [b_0^*, b_n^*]$*, any choice* $b' \in \left(b_n^*, b_{n+1}^*\right)$ *can be improved by* $b' = b_n^*$*.*

Proof. Since the tax rate is constrained when $b \geq b_0^*$, the government's objective function follows (71). For $b' \geq b_0^*$, differentiating V with respect to b' yields

$$\frac{\partial V}{\partial b'} = \frac{(1+\psi)\,\theta}{b' - Rb + \bar{\tau}w} - \frac{\theta\,(1+\lambda)\,\beta R}{b_n^* - Rb' + \bar{\tau}w}.$$

It is sufficient to prove that $\frac{\partial V}{\partial b'} \leq 0$ at $b' = b_n^*$ for $b \in [b_0^*, b_n^*]$.

$$\left.\frac{\partial V}{\partial b'}\right|_{b'=b_n^*} = \frac{(1+\psi)\,\theta}{b_n^* - Rb + \bar{\tau}w} - \frac{\theta\,(1+\lambda)\,\beta R}{b_n^* - Rb_n^* + \bar{\tau}w} \leq 0$$

We show that

$$\begin{aligned}
\frac{(1+\psi)}{b_n^* - Rb + \bar{\tau}w} &\leq \frac{(1+\lambda)\,\beta R}{b_n^* - Rb_n^* + \bar{\tau}w} \\
(1+\psi)\,(b_n^* + \bar{\tau}w) - (1+\psi)\,Rb_n^* &\leq (1+\lambda)\,\beta R\,(b_n^* + \bar{\tau}w) - (1+\lambda)\,\beta R^2 b \\
(1+\psi - (1+\lambda)\,\beta R)\,(b_n^* + \bar{\tau}w) &\leq (1+\psi - (1+\lambda)\,\beta R)\,Rb_n - (1+\lambda)\,\beta R^2\,(b - b_n^*) \\
((1+\lambda)\,\beta R - 1 - \psi)\,(b_n^* - Rb_n^* + \bar{\tau}w) &\geq (1+\lambda)\,\beta R^2\,(b - b_n^*)
\end{aligned}$$

This is always true if $b \leq b_n^*$. ■

Lemma 12 *Suppose that future policy outcomes follow (64), (65) and (66). Then, for any* $b \in \left[b_n^*, b_{n+1}^*\right]$, $b' = b_{n+s}$ *for any* $s > 0$ *is dominated by* $b' = b_n^*$.

Proof. We show that for any $b \in \left[b_n^*, b_{n+1}^*\right]$,

$$\begin{aligned}
&\theta\,(1+\psi)\log(b_n^* - Rb + \bar{\tau}w) + \theta\,(1+\lambda)\,\beta\log(b_n^* - Rb_n^* + \bar{\tau}w) \\
> \;&\theta\,(1+\psi)\log\left(b_{n+s}^* - Rb + \bar{\tau}w\right) + \theta\,(1+\lambda)\,\beta\log\left(b_{n+s} - Rb_{n+s}^* + \bar{\tau}w\right). \qquad (72)
\end{aligned}$$

Rearrange

$$\begin{aligned}
&\theta\,(1+\psi)\left(\log(b_n^* - Rb + \bar{\tau}w) - \log\left(b_{n+s}^* - Rb + \bar{\tau}w\right)\right) \\
> \;&\theta\,(1+\lambda)\,\beta\left(\log\left(b_{n+s}^* - Rb_{n+s}^* + \bar{\tau}w\right) - \log(b_n^* - Rb_n^* + \bar{\tau}w)\right)
\end{aligned}$$

The LHS and RHS of this expression can be written as

$$\sum_{i=0}^{i=s-1} \theta\,(1+\psi)\left(\log\left(b_{n+i}^* - Rb + \bar{\tau}w\right) - \log\left(b_{n+i+1}^* - Rb + \bar{\tau}w\right)\right) \qquad (73)$$

$$\sum_{i=0}^{i=s-1} \theta\,(1+\lambda)\,\beta\left(\log\left(b_{n+i+1}^* - Rb_{n+i+1}^* + \bar{\tau}w\right) - \log\left(b_{n+i}^* - Rb_{n+i}^* + \bar{\tau}w\right)\right). \qquad (74)$$

According to the difference equation

$$\left(b_n^* - Rb_{n+1}^* + \bar{\tau}w\right)^{1+\psi}(b_n^* - Rb_n^* + \bar{\tau}w)^{(1+\lambda)\beta} = \left(b_{n+1}^* - Rb_{n+1}^* + \bar{\tau}w\right)^{1+\psi+(1+\lambda)\beta}.$$

(74) is equal to

$$\sum_{i=0}^{i=s-1} \theta\left(1+\psi\right)\left(\log\left(b_{n+i}^{*}-Rb_{n+i+1}^{*}+\bar{\tau}w\right)-\log\left(b_{n+i}^{*}-Rb_{n+i}^{*}+\bar{\tau}w\right)\right)$$

Due to the increasing b_n^*,

$$\begin{aligned} & \log\left(b_{n+i}^{*}-Rb+\bar{\tau}w\right)-\log\left(b_{n+i+1}^{*}-Rb+\bar{\tau}w\right) \\ > \ & \log\left(b_{n+i}^{*}-Rb_{n+i+1}^{*}+\bar{\tau}w\right)-\log\left(b_{n+i+1}^{*}-Rb_{n+i+1}^{*}+\bar{\tau}w\right) \\ > \ & \log\left(b_{n+i}^{*}-Rb_{n+i+1}^{*}+\bar{\tau}w\right)-\log\left(b_{n+i}^{*}-Rb_{n+i}^{*}+\bar{\tau}w\right) \end{aligned}$$

for any $b \in \left[b_n^*, b_{n+1}^*\right]$. This establishes that the LHS of (72) is indeed larger than the RHS of (72). ■

Lemma 13 *Suppose that future policy outcomes follow (64), (65) and (66). Then, for* $b \in [\underline{b}, b_0^*]$, $b' = b_s^*$ *for any* $s > 0$ *is dominated by* $b' = b_0^*$.

Proof. We need to show that for any $b \in \left[b_n^*, b_{n+1}^*\right]$,

$$\begin{aligned} & \left(1+\beta+\theta\left(1+\psi\right)\right)\log\left(b_0^*-Rb+w\right)+\theta\left(1+\lambda\right)\beta\log\left(b_0^*-Rb_0^*+\bar{\tau}w\right) \\ > \ & \left(1+\beta+\theta\left(1+\psi\right)\right)\log\left(b_s^*-Rb+w\right)+\theta\left(1+\lambda\right)\beta\log\left(b_s^*-Rb_s^*+\bar{\tau}w\right). \end{aligned}$$

The rest of the proof simply follows the same procedure as in Lemma 12. ■

Lemma 14 *Suppose* $z_H > z_L$ *and* $b_H > b_L$. *Then,*

$$\begin{aligned} & \theta\left(1+\psi\right)\log\left(z_H-Rb_L+\bar{\tau}w\right)+\theta\left(1+\lambda\right)\beta\log\left(B\left(z_H\right)-Rz_H+\bar{\tau}w\right) \\ > \ & \theta\left(1+\psi\right)\log\left(z_L-Rb_L+\bar{\tau}w\right)+\theta\left(1+\lambda\right)\beta\log\left(B\left(z_L\right)-Rz_L+\bar{\tau}w\right) \end{aligned}$$

implies that

$$\begin{aligned} & \theta\left(1+\psi\right)\log\left(z_H-Rb_H+\bar{\tau}w\right)+\theta\left(1+\lambda\right)\beta\log\left(B\left(z_H\right)-Rz_H+\bar{\tau}w\right) \\ > \ & \theta\left(1+\psi\right)\log\left(z_L-Rb_H+\bar{\tau}w\right)+\theta\left(1+\lambda\right)\beta\log\left(B\left(z_L\right)-Rz_L+\bar{\tau}w\right). \end{aligned}$$

Proof. We need to show that

$$\begin{aligned} \log\left(z_H-Rb_L+\bar{\tau}w\right) &> \log\left(z_L-Rb_L+\bar{\tau}w\right) \Rightarrow \\ \log\left(z_H-Rb_H+\bar{\tau}w\right) &> \log\left(z_L-Rb_H+\bar{\tau}w\right) \end{aligned}$$

Define

$$F\left(b\right) \equiv \log\left(z_H-Rb+\bar{\tau}w\right)-\log\left(z_L-Rb+\bar{\tau}w\right)$$

It is straightforward that F is increasing in b since $z_L < z_H$. Hence, if $F\left(b_L\right) > 0$, $F\left(b_H\right)$ must be positive. ■

Lemma 15 *Suppose $z_H > z_L$ and $b_H > b_L$. Then,*

$$\begin{aligned} & \left(1+\beta+\theta\left(1+\psi\right)\right)\log\left(z_H - Rb_L + \bar{\tau}w\right) + \theta\left(1+\lambda\right)\beta\log\left(B\left(z_H\right) - Rz_H + \bar{\tau}w\right) \\ > \ & \left(1+\beta+\theta\left(1+\psi\right)\right)\log\left(z_L - Rb_L + \bar{\tau}w\right) + \theta\left(1+\lambda\right)\beta\log\left(B\left(z_L\right) - Rz_L + \bar{\tau}w\right) \end{aligned}$$

implies that

$$\begin{aligned} & \theta\left(1+\psi\right)\log\left(z_H - Rb_H + \bar{\tau}w\right) + \theta\left(1+\lambda\right)\beta\log\left(B\left(z_H\right) - Rz_H + \bar{\tau}w\right) \\ > \ & \theta\left(1+\psi\right)\log\left(z_L - Rb_H + \bar{\tau}w\right) + \theta\left(1+\lambda\right)\beta\log\left(B\left(z_L\right) - Rz_L + \bar{\tau}w\right). \end{aligned}$$

The proof follows the same procedure as in Lemma 14.

Now we can prove the proposition. We start with $b \in [\underline{b}, b_0^*]$, Lemma 10 and 13 imply that the optimal $b' = b_0^*$. Then we move to $b \in [b_0^*, b_1^*]$, Lemma 11 and 12 establish that $b' = b_0^*$ is better than any $b' \geq b_0^*$. Moreover, Lemma 15 establishes that any choice $b' < b_0^*$ cannot be optimal. So the optimal solution $b' = b_0^*$. The proof is completed by following the same procedure for any $n \geq 1$ (using Lemma 14).

C.2 Calibrated Economy with Different ω's.

In Figure A2, we show the policy function $B(b)$ for simulated economies corresponding to the calibration of Table 1 and ω changes between 0.5 and 1 (to ease visualization we only report three simulations with $\omega = 0.5$, 0.7 and 0.9). The figure shows that B(b) changes "with continuity" as we vary ω.

FIGURE A2 HERE

C.3 Linear Equilibrium with Political Uncertainty.

Proposition 7 *Assume that $\xi = 0$ and $\lambda = 0$. Then, the equilibrium with political shocks is given by the following policy functions.*

$$\begin{aligned} T\left(b,p\right) &= 1 - \frac{\left(1-\omega\right)R\left(1+\beta\right)}{w\left(\left(1-\omega\right)\left(1+\theta\left(1+p\right)\right)\left(1+\beta\right)+\omega\theta\left(1+p\right)\right)}\left(\bar{b}-b\right), \\ G\left(b,p\right) &= \frac{\theta\left(1+p\right)R}{\omega\theta\left(1+p\right)+\left(1-\omega\right)\left(1+\theta\left(1+p\right)\right)\left(1+\beta\right)}\left(\bar{b}-b\right), \\ B\left(b,p\right) &= \bar{b} - \frac{\left(1-\omega\right)\theta\left(1+p\right)\beta R}{\omega\theta\left(1+p\right)+\left(1-\omega\right)\left(1+\theta\left(1+p\right)\right)\left(1+\beta\right)}\left(\bar{b}-b\right), \end{aligned}$$

where $\bar{b} \equiv w/\left(R-1\right)$, and $p \in \{p_r, p_l\}$.

PROOF. When $\xi = 0$ and $\lambda = 0$, the GEE, (35), simplifies to:

$$\frac{1}{G(b,p)} = -\frac{\beta}{1+\psi} \cdot E_p \left[\frac{G'(B(b,p),p')}{G(B(b,p),p')} \right], \tag{75}$$

We guess that

$$G(b,p) = \gamma(p)\left(\bar{b} - b\right). \tag{76}$$

Combining equations (58), (60), and (76) imply that

$$\bar{b} - B(b,p) = \left(R - \left(1 + \frac{1+\beta}{(1+p)\,\theta\,(1+\psi)} \right) \gamma(p) \right) \left(\bar{b} - b\right). \tag{77}$$

Combining equations (75)-(77) and rearranging terms yield;

$$\gamma(p) = \frac{(1+\psi)\,\theta\,(1+p)\,R}{\psi\theta\,(1+p) + (1+\beta)\,(1+\theta\,(1+p))}$$

and

$$G(b,p) = \frac{(1+\psi)\,\theta\,(1+p)\,R}{\psi\theta\,(1+p) + (1+\beta)\,(1+\theta\,(1+p))} \left(\bar{b} - b\right). \tag{78}$$

Hence, substituting the expression of $\gamma(p)$ into (77) leads to

$$B(b,p) = \bar{b} - \frac{\beta R \theta\,(1+p)}{\psi\theta\,(1+p) + (1+\beta)\,(1+\theta\,(1+p))} \left(\bar{b} - b\right). \tag{79}$$

Additionally, in the case of $\lambda = \xi = 0$ the intra-temporal condition, (58), simplifies to

$$1 - T(b,p) = \frac{1+\beta}{(1+\psi)\,\theta\,(1+p)\,w} G(b,p). \tag{80}$$

Finally, recall that, when $\lambda = 0$, then $\psi = \omega/(1-\omega)$. Then, equations (78), (79) and (80) yield the policy functions in Proposition 7. This concludes the proof.